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AND

DISCUSSIONS

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Note.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

*** Publication of closing discussion pending.**

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PAPERS

DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

RAINBOW ARCH BRIDGE OVER NIAGARA GORGE

A SYMPOSIUM

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **March 1, 1944.**

FOREWORD[1a]

It is appropriate that the engineers responsible for the construction of the longest fixed arch span built to date should give the engineering profession a record of the design and erection of that monumental structure.

The selection of the proper bridge to be built at this crossing involved not only the economics of materials and erection but also that elusive item, the economics of appearance. The problem of esthetics was solved in this case by selecting the graceful arch structure for the main span, and approach spans which, in the opinion of the architect, harmonized with the surrounding structures particularly on the Canadian side.

Some of the problems encountered by Mr. Hardesty in his development of the design are discussed in the opening paper. While making his studies relating to the design, he became convinced that additional information was needed to guide engineers in the design of arch ribs. As a result, the problem was presented to Professor Timby, who supervised and directed studies on two-hinged arch models. These studies comprise the subject matter of the second paper in this Symposium.

The third paper of the Symposium gives a record of some of the problems and studies necessary to the construction of such a structure. The mathematical calculations necessary for positive control of such a structure during erection are very extensive. However, one who has seen the close check between the mathematical prediction and the measured forces in such a structure as this will have a greater confidence in future designs of statically indeterminate structures.

The fourth and last paper in the Symposium presents the solution to the many problems in the erection of a monumental bridge and describes the erection procedure.

Notation.—The following letter symbols, used in this Symposium, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932), prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

A = area; cross-sectional area of the arch rib:
- A_a = cross-sectional area of rib at point A;
- A_w = web area;

b = a subscript denoting base arch;
C = coefficient: C_1, C_2, C_3, and C_4 are empirical coefficients in Eq. 24;
$\overline{DL}$ = dead load; $\overline{DL}_b$ = dead load on the base arch;
E = Young's modulus of elasticity:
E_b = Young's modulus of elasticity referring to the base arch;
E_s = modulus of elasticity in shear;

[1a] Material in the "Foreword" is not open to discussion. Suggestions for correction or amendments should be transmitted to the Editor for possible inclusion in the official *Transactions* printing of the "Foreword."

e' = accidental errors; maximum probable ordinate of a departure;
F = a force;
H = horizontal force, or component of force:
- H_b = a horizontal force required, with V_b, to balance a system;
- H_c = thrust due to camber;
- H_r = (see V_r);
- H_t = change in thrust due to temperature change t;
- H_1 = horizontal force required to balance the cable;
- H' = differential force;
- ΔH = a horizontal increment of force resulting from the change in geometry of the rib;

h = a subscript denoting "horizontal shear";
I = moment of inertia of the rib cross section:
- I_b = moment of inertia referred to the base arch;

$\bar{I}$ = impact load;
L = span length:
- L_b = span length of base arch;
- ΔL = increments of span length;

$\overline{LL}$ = live load;
l = length of the horizontal projection of the half arch;
M = bending moment:
- M_a = moment at point A;
- M_b = bending moment in the base arch;
- M_e = moment by usual elastic analysis;
- M_m = total bending moment at point m;
- M' = simple beam bending moment;
- Δm = differences between bending moments;

N = factor of safety;
P = a concentrated force;
Q = a bending moment used in Table 5;
R = a ratio, $\dfrac{\overline{DL}}{\overline{LL}}$;
r = radius of curvature of the arch rib: r_o = original radius;
S = section modulus;
s = unit stress; s_y = elastic limit;
T = total direct thrust:
- T_a = total axial thrust at the point A;
- T_t = thrust due to temperature effect;
- T_w = thrust due to wind;

t = change in temperature, in degrees Fahrenheit;
u = radial component of displacement of the arch rib:
- u_a = radial component at point A;
- u_e = radial component of displacement computed by the elastic theory;
- u_f = radial component of displacement due to force F;
- u_m = radial displacement at point m;

V = vertical shear:

V_b = "basic" vertical shear;

V_r = vertical shear which, with H_r, it is assumed would raise the rib to point A;

V_1 = vertical force required to balance the cable;

v = a subscript denoting "vertical shear";

W = a total unit load, $\overline{DL} + \overline{LL} + \bar{I}$:

W_w = working load, ordinary load;

W_{yp} = load W that produces a yield-point stress in the arch rib;

X_c = centroid ordinate;

Y_c = centroid ordinate;

y = vertical ordinate of arch rib: y_a = ordinate at point A (nominal outline);

α = slope angle of the arch rib at any point;

Δ = horizontal deflection;

δ = deflections due to thrust, bending, shear, camber, or temperature:

δ_h = horizontal deflection;

δ_v = vertical deflection;

δ' = deformation produced by a load of 1 kip;

ϵ = linear coefficient of temperature expansion;

η = vertical displacement;

θ = angle change:

θ_m = total angle changes;

$\Delta\theta$ = small increment of angle change;

ρ = horizontal displacement;

τ = change in angles in relation to adjacent members, due to bending:

τ_m = angle change at point m; and

ϕ = angle change: ϕ_c = angle change at the crown.

Acknowledgments.—The bridge was built under the direction of the Niagara Falls Bridge Commission, a body created by authority of an act of Congress.

The consultants retained by the Commission were: Waddell & Hardesty of New York, N. Y., who prepared the designs and plans; the Edward P. Lupfer Corporation of Buffalo, N. Y., who supervised the construction; and Aymar Embury II, M. Am. Soc. C. E., of New York who was consulting architect.

The deformeter analysis of the superstructure, mentioned in the first paper of the Symposium, was made by Egbert Hardesty, Jun. Am. Soc. C. E., and was reported fully in a thesis entitled "Model Analysis of the Proposed Rainbow Arch Bridge Over the Niagara River," which was presented to Rensselaer Polytechnic Institute in Troy, N. Y., in 1941, in partial fulfilment of the requirements for the degree of Bachelor of Civil Engineering.

The work described in the second paper was done at Princeton University in Princeton, N. J., under the sponsorship of the School of Engineering, of which Kenneth H. Condit is Dean of Engineering and Philip Kissam, M. Am.

Soc. C. E., is chairman of the Department of Civil Engineering. Complete results of these tests, including considerable material not touched on in the Symposium paper, are presented in the form of a thesis entitled "Moments in Flexible Arches," submitted to Princeton University in June, 1941, jointly by Messrs. McEldowney and Mead in partial fulfilment of the requirements for the degree of Civil Engineer. John W. Graham, Jr., Jun. Am. Soc. C. E., made the preliminary design for the loading frame and assisted in its construction. The authors of the first Symposium paper gave many valuable suggestions during the planning of this investigation and in the preparation of the digest of the results.

Award of the general contract for the fabrication and erection of the 950-ft main arch span of the Rainbow Bridge, including the concrete deck, was made to the Bethlehem Steel Company early in May, 1940. The ribs of the Rainbow arch were fabricated in the United States by the Bethlehem Steel Company and the remainder was fabricated in Canada under a contract let to the Canadian Bridge Company. The latter, in turn, sublet the fabrication of the spandrel girders to the Hamilton Bridge Company. In the construction operations, it was required that all workmen employed should be residents of the country where such operations were being done. It was desirable, however, that the field work be under one management, and it was agreed that all construction operations would be by the Bethlehem Steel Company.

The entire erection scheme was worked out under the direction of D. S. Gendell, Jr., M. Am. Soc. C. E., General Manager of Erection for the Bethlehem Steel Company. The author of the last paper of the Symposium, Mr. Durkee, was in charge of the erection of the steelwork as Resident Engineer, assisted by W. W. Oskin, J. T. York, and W. H. Thorn as junior engineers. R. S. Smith was foreman in charge on the New York side of the river and D. S. Shuman on the Canadian side.

The McLain Construction Corporation of Buffalo was contractor for the concrete approaches on the New York side and subcontractor for the concrete deck on the United States half of the steel arch. On the Canadian side, Aiken and MacLachlan, Ltd., of St. Catherines, Ontario, were contractors for the concrete approaches and subcontractors for the concrete deck.

DESIGN

By Shortridge Hardesty[1] and J. M. Garrelts,[2] Members, Am. Soc. C. E., and I. G. Hedrick, Jr.,[3] Jun. Am. Soc. C. E.

Synopsis

The first part of this paper treats the design of the fixed arch bridge spanning the gorge between Niagara Falls, N. Y., and Niagara Falls, Ont., Canada. The design loads and allowable unit stresses are given, together with the stresses as calculated by the conventional elastic theory.

In the second part, the effects of rib deflections are discussed. The resulting moments at the quarter point are evaluated for a preliminary, two-hinged arch rib, and for the final fixed arch rib as built. A relatively simple procedure is proposed for calculating approximate values of such moments. These results can be used for preliminary studies, and in many cases are sufficiently accurate for use in final design.

PART I.—DESIGN

Introduction

The new Niagara arch bridge (Fig. 1) spanning the Niagara gorge is about one-half mile downstream from the Falls and 400 ft north of the location of the

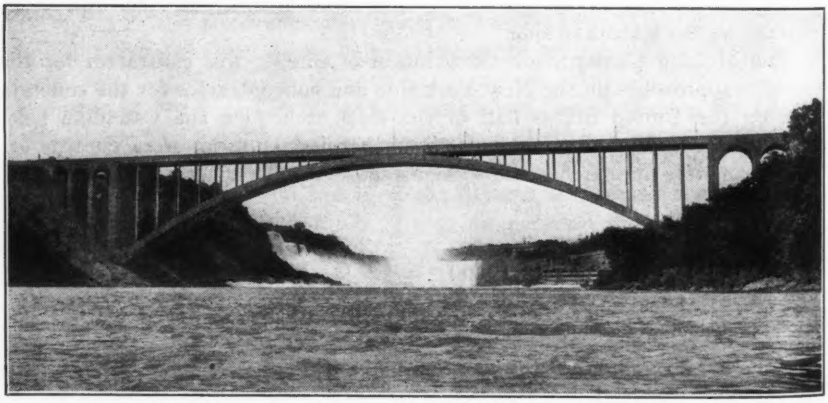

Fig. 1.—View of Arch

former International Falls View arch which it has replaced. The previous structure (wrecked by river ice on January 27, 1938) was a two-hinged arch

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with an 840-ft span. Each rib of this structure consisted of a truss with chords 26 ft center to center, and with a rise of 150 ft. In accordance with present-day standards, the spandrel columns and the truss members were of light design.

At the site of the new bridge, the gorge is 1,250 ft wide at the rim and about 830 ft wide at the surface of the water. The depth from the rim to mean water level is 180 ft, which, with a water depth of 175 ft, makes a total of 355 ft. The walls of the gorge on both sides of the river drop almost vertically for a distance of 80 ft at the top, and then slope down at approximately 45° to the water line. The talus covering on the slopes is from 10 ft to 35 ft thick. The vertical cliffs are of dolomite, and the rock beneath the talus slopes consists of Rochester shale, Irondequoit limestone, Wolcott limestone, Sodus shale, Thorold sandstone, and Albion shale and sandstone. All strata are practically horizontal. A cross-sectional view of the various strata is shown in Fig. 2, a further description being as follows:

Clinton Formation.—

Rochester shale is rather soft on top. It weathers badly, but is a good firm rock requiring blasting where unexposed. It has irregular horizontal and vertical seams.

Irondequoit limestone is hard, firm, durable, and deeply bedded.

Wolcott limestone is a durable rock which does not occur in such a heavy strata as the Irondequoit limestone.

Sodus shale is bluish in color and is firm where not exposed.

Albion Sandstone Formation.—

Thorold sandstone is white, hard, and durable.

The Albion shale and sandstone stratum is a rather soft red sandstone interlaid with reddish shale. The stratum directly beneath is a much harder gray and red sandstone, which on the east shore is mottled, with thinner shale seams than in the overlying shale and sandstone stratum.

Preliminary Studies

Considerable thought was given to the esthetics of the new bridge and to the creation of a structure that would blend with the natural beauty and grandeur of the Niagara gorge. Architectural studies and preliminary designs showed definite advantages in favor of an arch-type structure for this location. In addition to its esthetic superiority, the arch is structurally appropriate; and the inclined rock walls provide a perfect support for the arch abutments. The approach structure favored by the Bridge Commission was one with massive concrete shafts and arches. This contrast in type of construction served to define the main arch span (Figs. 1 and 3).

The plate girder rib type was selected, rather than the trussed rib or the spandrel braced arch, because of its architectural simplicity. Transverse and longitudinal bracing between the spandrel columns was avoided in order to maintain the clean-cut lines of the structure (see Fig. 4). Horizontal forces are transferred from the floor to the plane of the arch through shear in the

columns, except at two points near the center where inconspicuous sway bracing was used.

The effects of the participation of the deck with the superstructure, and of expansion joints in the deck floor system, were investigated by a deformeter analysis, using a celluloid model of one rib with spandrel columns and spandrel girders. This analysis demonstrated that the joints for the spandrel structure

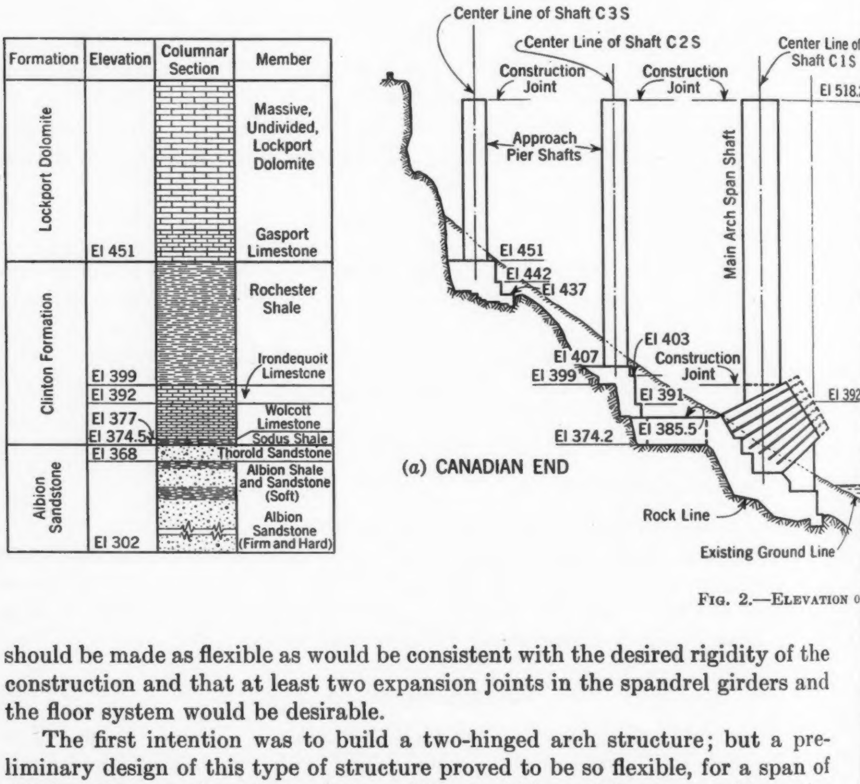


Fig. 2.—Elevation of

should be made as flexible as would be consistent with the desired rigidity of the construction and that at least two expansion joints in the spandrel girders and the floor system would be desirable.

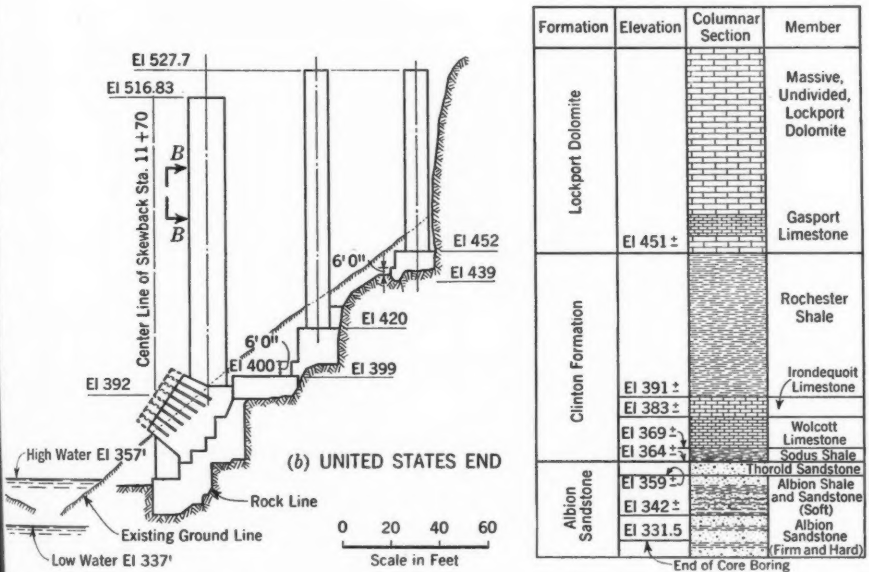
The first intention was to build a two-hinged arch structure; but a preliminary design of this type of structure proved to be so flexible, for a span of this length, as to show that a design which would be satisfactory would require ribs of excessive moment of inertia and sectional areas. The hingeless type then was investigated, and it was found that a rib with considerably less area and moment of inertia would be safe, and would be subject to much smaller deflections. Since both economy and rigidity favored the hingeless type, it was selected.

Design Loads and Allowable Unit Stresses

The live load used for design was the H-20 loading of the 1935 Standard Specifications for Highway Bridges of the American Association of State Highway Officials. For the design of the arch rib, this is equivalent to a uniform live panel load of 54.4 kips and a concentrated live panel load of 26.7 kips, the

panel length being 39.4 ft. The impact effect was calculated in accordance with the forementioned specifications. The intensity of the wind force was assumed to be 30 lb per sq ft on one-and-a-half times the projected area of the structure in a vertical plane, plus 200 lb per lin ft applied 6 ft above the roadway.

The design provided for a temperature variation of $\pm 60°$ F from an arbitrary normal of 50° F. Allowance also was made for a possible error of ± 1 in.



South Piers and Shafts

in span length. Such an error would produce stresses equivalent to those due to a change in temperature of $\pm 15°$ F.

The allowable unit stresses and the requirements for the details of design are in accordance with the 1938 Specifications for Steel Railway Bridges of the American Railway Engineering Association (A.R.E.A.). The basic unit stresses, in pounds per square inch, were:

	Carbon	Silicon
Tension, net section	18,000	24,000
Compression, gross section	$15{,}000 - \frac{1}{4}\left(\frac{l}{r}\right)^2$	$20{,}000 - 0.46\left(\frac{l}{r}\right)^2$

The allowable unit stresses for the silicon steel arch ribs, for use with stresses computed by the elastic theory, in pounds per square inch, were:

Dead load+live load+impact load+temperature+error............20,000
Dead load+live load+impact load+temperature+error+wind load..25,000

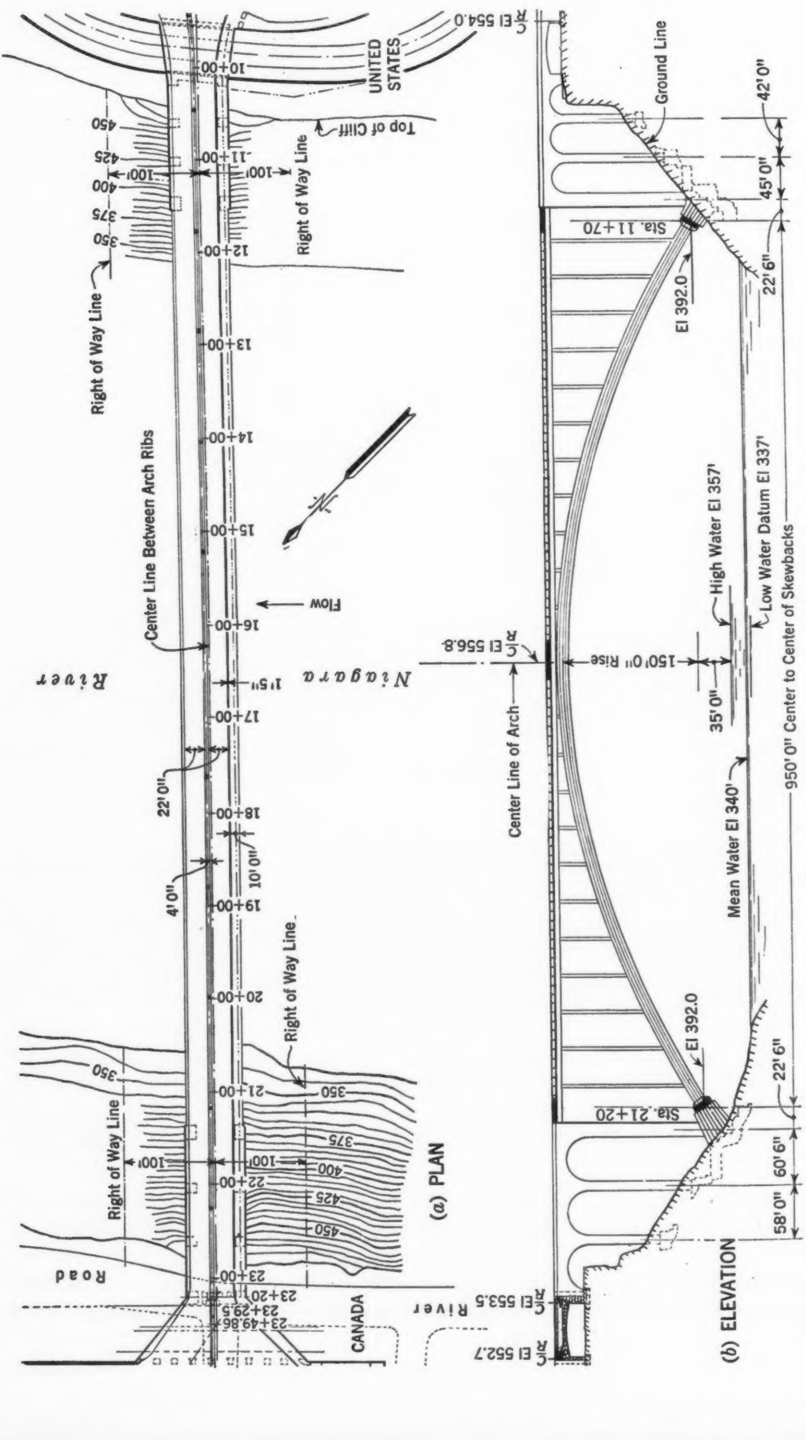


FIG. 3.—PLAN AND ELEVATION OF SUPERSTRUCTURE

Design of Main Arch Span

Floor System.—The bridge carries two 22-ft roadways (see Fig. 5) separated by a 4-ft mall, and a 10-ft walkway on the south side facing the Falls. The roadway floor consists of $7\frac{1}{2}$-in. concrete slab reinforced with welded-bar reinforcing trusses, which is supported on 30-in. wide-flange 108-lb rolled-beam

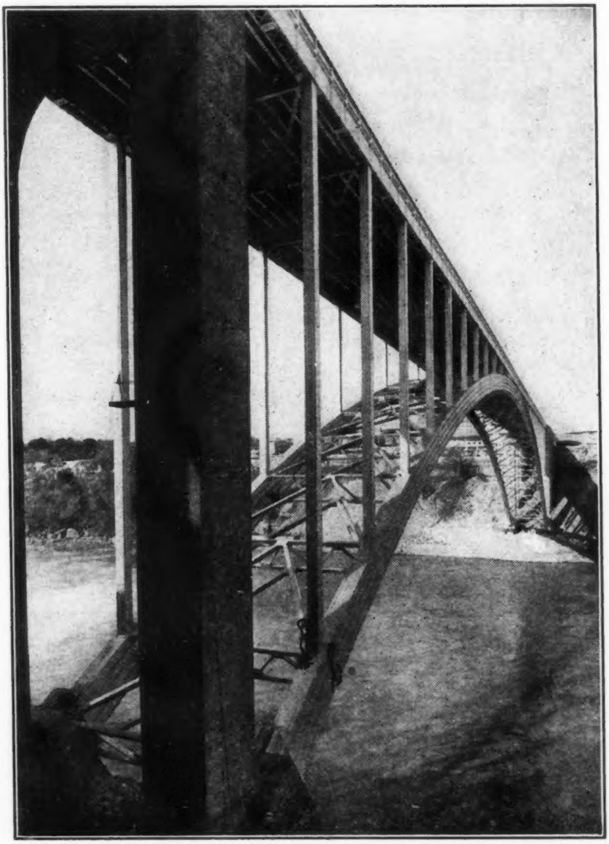


Fig. 4.—View of Spandrels and Wind Bracing

stringers, a maximum of 5 ft 5 in. apart. The walkway is a 4-in. concrete slab. The stringers frame into silicon steel floor beams 6 ft $0\frac{1}{2}$ in. in depth, 39 ft 4 in. apart. Box-type spandrel girders, 6 ft $0\frac{1}{2}$ in. in depth, consist of a top cover plate, two top-flange angles, two web plates, and two bottom-flange angles. The spandrel girders are connected to the columns at most points with short, flexible angle connections which allow some relative movement in the longitudinal direction. However, rigid connections were used at panel point 5 (see Fig. 6) in order to provide longitudinal stiffness in the floor system, and at the

long columns near the ends which were so flexible that rigid connections were found desirable.

Spandrel Columns.—The spandrel columns are rectangular box sections. They were designed for dead load, live load, wind loads, and stresses due to

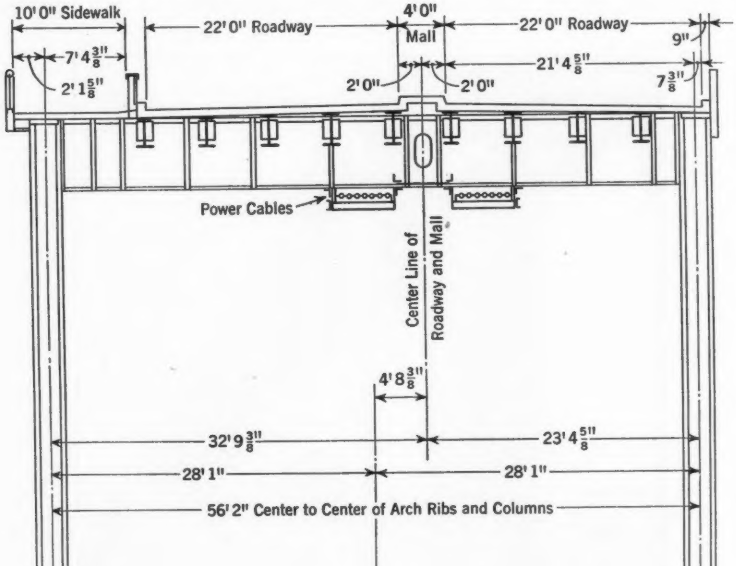


FIG. 5.—CROSS SECTION OF DECK

participation effects. Shears and moments are produced in the columns when the supporting arch ribs rotate or are displaced. Transverse wind loads also produce shears and moments in the columns, due to unequal transverse dis-

placements of the floor system and arch ribs. The base of each column was milled to a plane, tangent to the curved top cover plate of the arch rib; and high restraining moments in the connections for the columns were reduced by using flexible base connections, the connection angles having sufficient spring to allow the columns to rock slightly in the longitudinal direction on the arch rib. The tops of the columns at panel point 5 were fixed to the spandrel girders, as has been stated previously.

Wind loads on the deck are carried partly by the roadway lateral system and partly by shear in the spandrel columns. The part of the load carried by each was determined by distributing the loads to the laterals and to the columns in such a manner that the resulting deflections would be consistent. An expansion joint was provided in the deck and the floor lateral system at panel point 3.

Laterals.—The arch rib laterals consist of a plane of K-type bracing at both the top and bottom flanges of the rib, with transverse sway frames at each panel point. These laterals are designed for the wind forces acting directly on the rib and also for the wind forces transferred from the deck to the rib through the spandrel columns.

Arch Ribs.—The arch ribs are silicon steel box girders 12 ft deep. Design loads and stresses are shown in Table 1. In addition to the stresses computed by the elastic analysis, stresses due to the deflection effects were calculated at certain points. The resulting effect of these secondary stresses is discussed subsequently. Each arch rib, except at the springings, has two 144-in. by $\frac{13}{16}$-in. web plates and eight 8-in. by 8-in. flange angles. Top and bottom cover plates 54 in. wide vary from $1\frac{1}{8}$ in. to $2\frac{1}{2}$ in. in thickness. A longitudinal diaphragm at the center of the web plates and longitudinal angles at the quarter points provide adequate stiffness to prevent buckling of the web plates between the normal diaphragms, which are placed at panel points and mid-panel points.

The rib splices were designed to transmit at least 60% of the total stress through the splice material. The abutting ends of the rib sections at the splices were faced accurately to secure an even bearing when assembled in the structure. For access to the rib sections, manholes were cut in the inner web plates, the area around the holes being reinforced to compensate for the area taken out. The web manholes are spaced at intervals of about 120 ft, and openings in the diaphragms permit longitudinal passage through the inside of the rib sections. Typical details for a section of the rib are shown in Figs. 7 and 8.

The make-up of Section 1 is as follows:

Main material	Details
Two web plates, 144 in. by $\frac{13}{16}$ in.	Two angles, 7 in. by 4 in. by $\frac{5}{8}$ in.
Two cover plates, 54 in. by $1\frac{1}{8}$ in.	Two angles, 4 in. by 4 in. by $\frac{1}{2}$ in.
One plate, 34 in. by $\frac{3}{4}$ in.	Four angles, 4 in. by 4 in. by $\frac{7}{16}$ in.
Eight angles, 8 in. by 8 in. by 1 in.	....
Four angles, 7 in. by 4 in. by $\frac{5}{8}$ in.	....

The make-up of Section 2 is identical except that $1\frac{1}{4}$-in. cover plates were used.

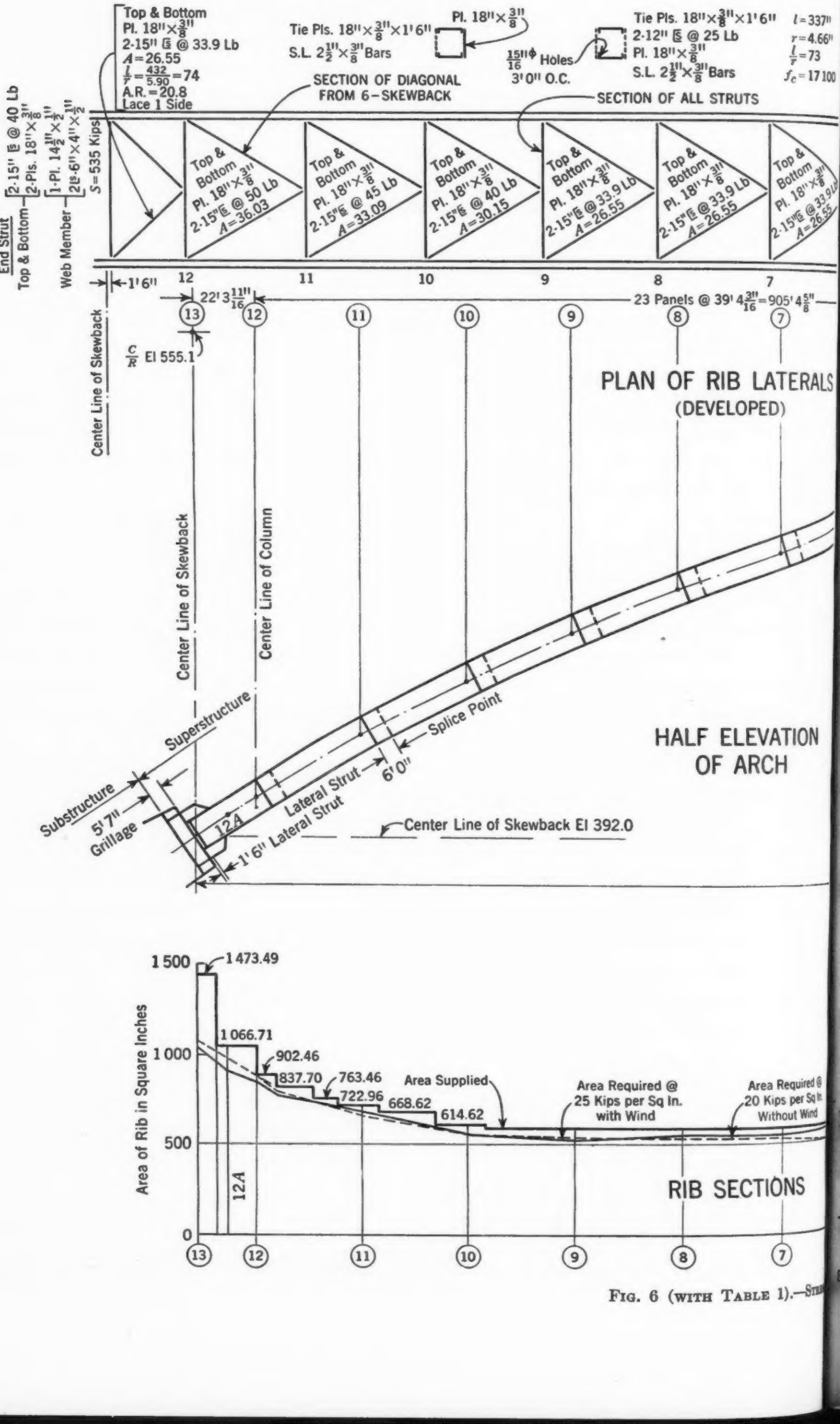
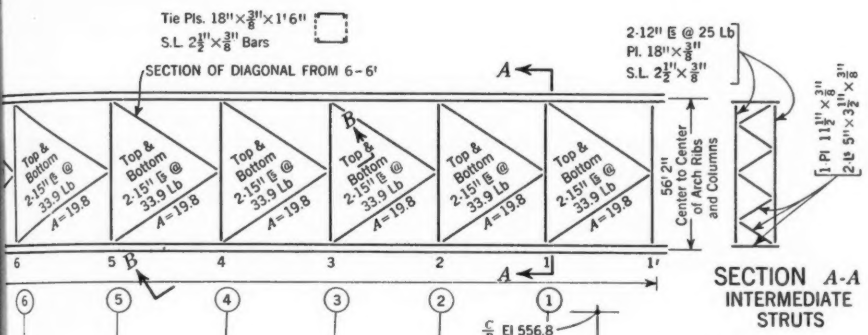
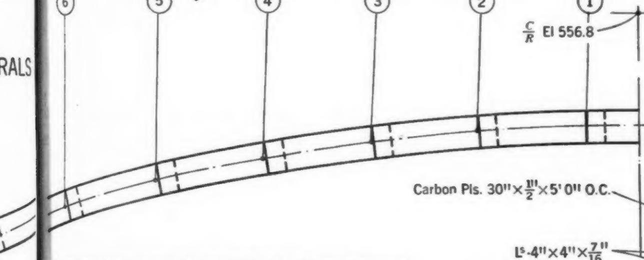
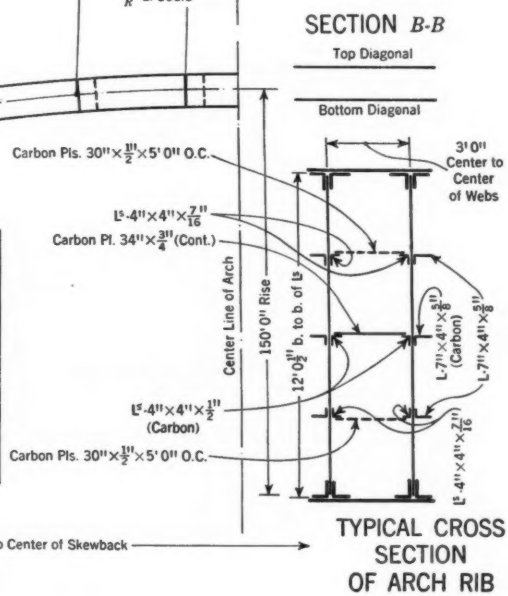


FIG. 6 (WITH TABLE 1).—STR

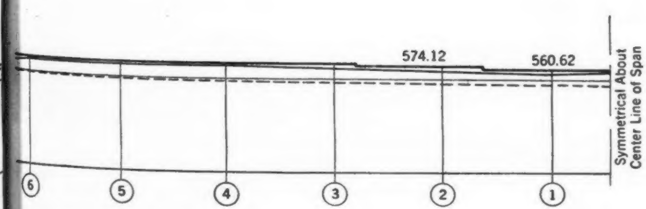






GEOMETRY OF ARCH RIB

PANEL POINT	HORIZONTAL DISTANCE FROM CROWN	VERTICAL DISTANCE FROM CROWN.	TANGENT OF AXIS WITH HORIZONTAL
1	19.682'	0.249'	0.0253
2	59.047	2.237	0.0758
3	98.412	6.219	0.1264
4	137.776	12.192	0.1772
5	177.141	20.194	0.2284
6	216.505	30.183	0.2802
7	255.870	42.249	0.3330
8	295.235	56.418	0.3871
9	334.599	72.749	0.4429
10	373.964	91.320	0.5010
11	413.328	112.232	0.5619
12	452.693	135.606	0.6263
13	475.000	150.000	0.6645



Sheets for Arch Ribs.

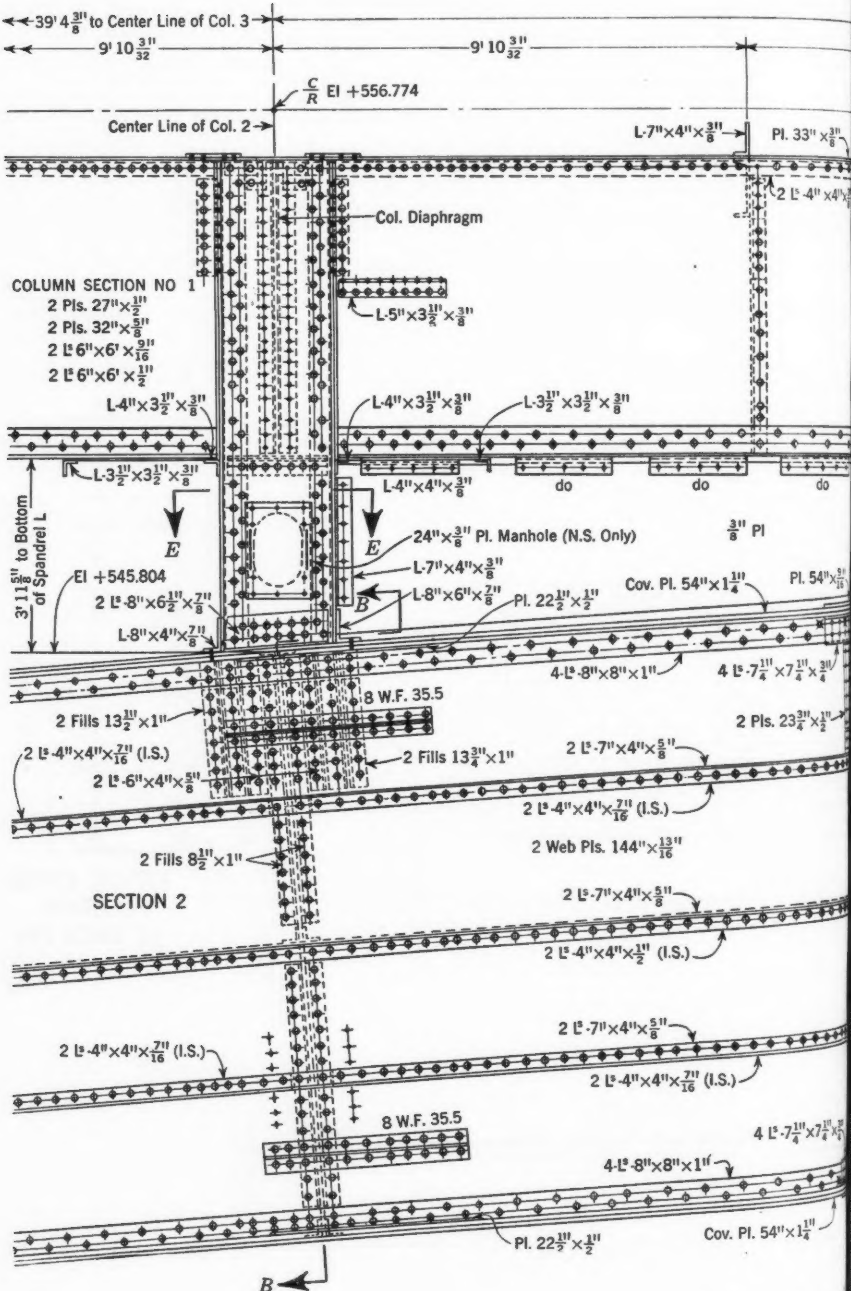
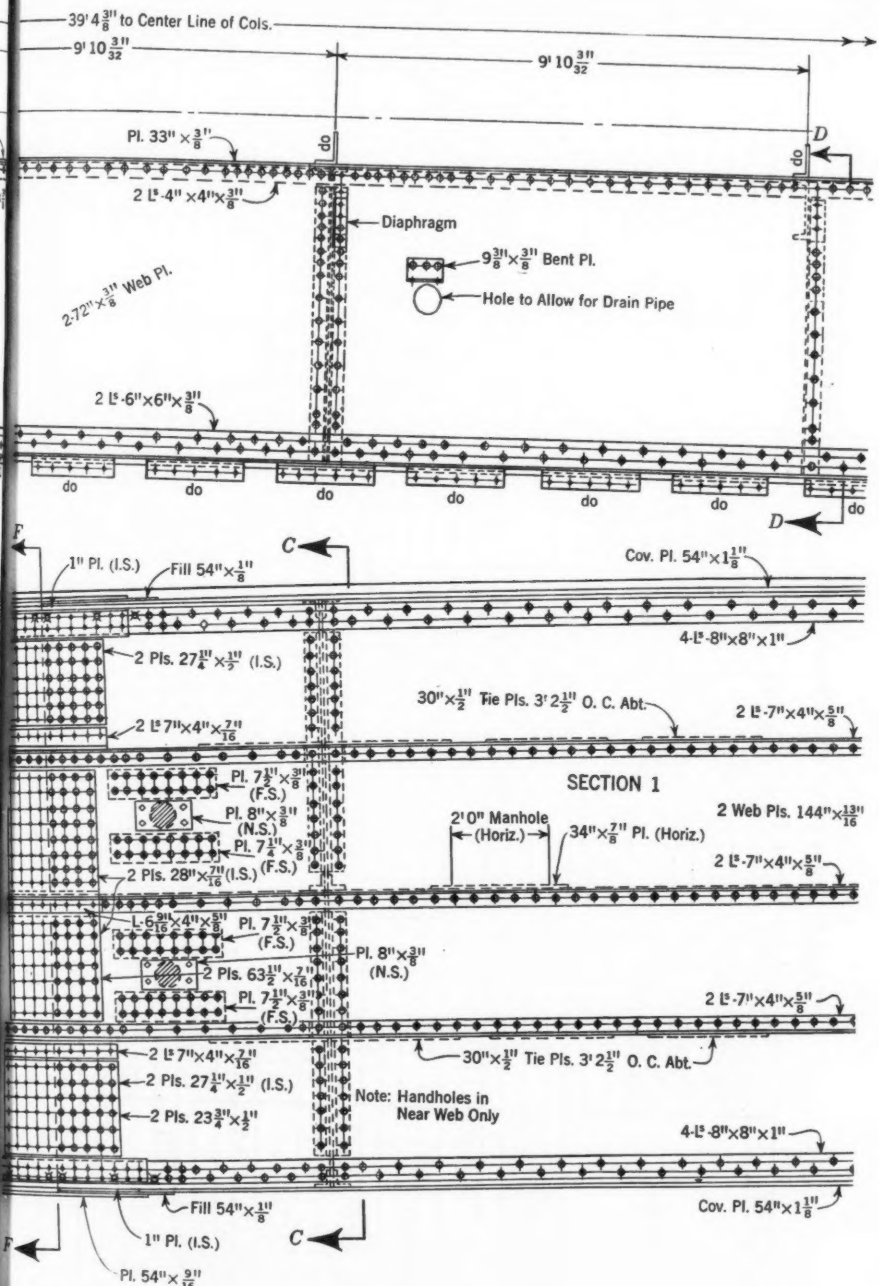


Fig. 7.—Detail of Ar



...Rib Sections 1 and 2

TABLE 1 (WITH FIG. 6).—STRESS SHEETS FOR ARCH RIBS

	POINT	SPRINGING		12 A (MIDWAY BET. 12 & 13)		12		11	
	SECTION	Grillage 12'×22'		2 Webs 144×13/16	: 234.00	2 Webs 144×13/16	: 234.00	2 Webs 144×13/16	: 234.00
		Area = 264□' = 38016□"		2 Webs 128×9/16 2 Webs 128×5/8	: 144.00 160.00	2 Webs 128×9/16	: 144.00	8Ls·8×8×1 1/8	: 133.84
				8Ls·8×8×1 1/8	: 133.84	8Ls·8×8×1 1/8	: 133.84	4 Cov Pls 54×1 1/4	: 270.00
		32 Anchor Bolts		4 Cov. Pls. 54×1 1/4	: 270.00	4 Cov Pls 54×1 1/4	: 270.00	6Ls·7×4×5/8	: 38 88
		3"ϕ Upset		2 Cov. Pls. 18×1 1/8	: 40.50	2 Cov. Pls 18×1 1/8	: 40.50	1·Pl. 34×3/4	: 25.50
		Area = 226.21□"		6Ls·7×4×5/8	: 38.88	6Ls·7×4×5/8	: 38 88	2Ls·4×4×1/2	: 7 50
				1·Pl. 33×3/4	: 24.75	1·Pl 34×3/4	: 25 50	4Ls·4×4×7/16	: 13 24
				2Ls·4×4×1/2	: 7.50	2Ls·4×4×1/2	: 7 50		
				4Ls·4×4×7/16	: 13 24	4Ls·4×4×7/16	: 13 24		
1	GROSS AREA - SQ. IN.				1,066.71		907.46		722 96
2	MOMENT OF INERTIA (IN.² FT.²)				22,138		20,621		17,810
3	SECTION MODULUS (IN.² FT.)	139,392			3,554		3,310		2,859
		THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT
4	D.L. (KIPS)	+7,172	-4,516	+7,160	-3,200 -3,484	+7,098	-852 -3484	+6,864	+489 -1274
5	L+I.(+M) "	+846	+28,850	+865	+25,500	+883	+21,940	+808	+12,480
6	L+I.(-M) "	+638	-27,690	+637	24,500	+636	-21,360	+666	-12,650
7	TEMPERATURE (T) "	±77	±9,818	±78	±9,180	±79	±8,484	±81	±6,318
8	ERROR (E) "	±19	±2,455	±20	±2,300	±20	±2,121	±20	+1,580
9	WIND (W) "	±2,865	±20,160	±2,860	±14,600	±2,381	±12,400	±1,943	±1,600
10	TOTAL STRESSES IN KIPS								
11	D+L+I+T+E (+M)	+8,114	+36,608	+8,123	+33,780	+8,080	+31,693	+7,773	+20,867
12	D+L+I+T+E (-M)	+7,714	-44,479	+7,699	-39,464	+7,635	-35,449	+7,429	-21,822
13	D+L+I+T+E+W (+M)	+10,979	+56,768	+10,983	+48,380	+10,461	+44,093	+9,716	-22,467
14	D+L+I+T+E+W (-M)	+4,859	-64,639	+4,839	-54,064	+5,254	-47,849	+5,486	-23,422
15	UNIT STRESS (K/Sq. IN.)	FROM +M	FROM -M	FROM +M	FROM -M	FROM +M	FROM -M	FROM +M	FROM -M
16	D+L+I+T+E								
17	f DIRECT	fc = 480	fc = 530	7.62	7.22	8.90	8 41	10 75	10 28
18	f BENDING	fs = 1,000	fs = 1,000	9.50	11.10	9 57	10 71	7 30	7 63
19	TOTAL			17.12	18 32	18.47	19.12	18 05	17 91
20	D+L+I+T+E+W								
21	f DIRECT	fc = 720	fc = 840	10.30	4.54	11.53	5.79	13 44	7 59
22	f BENDING	fs = 1,000	fs = 16,000	13.61	15.21	13 32	14 46	7 86	8 19
23	TOTAL			23.91	19.75	24 85	20.25	21 30	15 78

TABLE 1.—(*Continued*)

	10		9		8		7		6	
	2 Webs 144 × 13/16	= 234.00	2 Webs 144 × 13/16	= 234.00						
	8 Ls · 8 × 8 × 1	= 120.00	8 Ls · 8 × 8 × 1	= 120.00						
	2 Cov. Pls. 54 × 1 5/8	= 175.50	2 Cov. Pls. 54 × 1 3/8	= 148.50						
	6 Ls · 7 × 4 × 5/8	= 38.88	6 Ls · 7 × 4 × 5/8	= 38.88	←				Same as for Section 9	
	1 · Pl. 34 × 3/4	= 25.50	1 · Pl. 34 × 3/4	= 25.50						
	2 Ls · 4 × 4 × 1/2	= 7.50	2 Ls · 4 × 4 × 1/2	= 7.50						
	4 Ls · 4 × 4 × 7/16	= 13.24	4 Ls · 4 × 4 × 7/16	= 13.24						
1		614.62		587.62		587.62		587.62		587.62
2		13,721		12,701		12,701		12,701		12,701
3		2,229		2,070		2,070		2,070		2,070
	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT
4	+ 6,675	+793 -540	+ 6,519	+972 -280	+ 6,384	+ 1157 -38	+ 6,268	+ 1248 + 132	+ 6,178	+ 1181 + 197
5	+ 757	+ 6,380	+ 570	+ 5,590	+ 403	+ 9,170	+ 472	+ 11,760	+ 456	+ 13,400
6	+ 858	- 6,400	+ 1,103	- 5,460	+ 1,048	- 8,400	+ 894	- 11,050	+ 893	- 11,950
7	± 83	± 4,381	± 85	± 2,660	± 87	± 1,147	± 88	± 166	± 89	± 1,285
8	± 21	± 1,095	± 21	± 665	± 22	± 287	± 22	± 41	± 22	± 321
9	± 1,539	± 4,200	± 1,168	± 5,980	± 843	± 3,660	± 575	± 4,280	± 365	± 490
10										
11	+ 7,539	+ 12,649	+ 7,135	+ 9,887	+ 6,896	+ 11,761	+ 6,630	+ 13,215	+ 6,513	+ 16,187
12	+ 7,429	- 12,416	+ 7,456	- 9,065	+ 7,323	- 9,872	+ 7,272	- 11,125	+ 7,172	- 13,359
13	+ 5,917	+ 16,849	+ 5,967	+ 15,867	+ 6,053	+ 15,421	+ 6,055	+ 17,495	+ 6,148	+ 16,677
14	+ 8,968	- 16,616	+ 8,624	- 15,045	+ 8,166	- 13,532	+ 7,847	- 15,405	+ 7,537	- 13,849
15	FROM + M	FROM - M	FROM + M	FROM - M	FROM + M	FROM - M	FROM + M	FROM - M	FROM + M	FROM - M
16										
17	12.27	12.09	12.14	12.69	11.74	12.46	11.28	12.38	11.08	12.21
18	5.67	5.57	4.78	4.38	5.68	4.77	6.38	5.37	7.81	6.45
19	17.94	17.66	16.92	17.07	17.42	17.23	17.66	17.75	18.89	18.66
20										
21	9.76	14.59	10.15	14.68	10.30	13.90	10.30	13.35	10.46	12.83
22	7.56	7.45	7.66	7.27	7.45	6.53	8.45	7.44	8.06	6.69
23	17.32	22.04	17.81	21.95	17.75	20.43	18.75	20.79	18.52	19.52

TABLE 1.—(*Continued*)

	5		4		3		2		1	
							2 Webs 144 × 13/16	: 234.00	2 Webs 144 × 13/16	234.00
							8 Ls-8 × 8 × 1	: 120.00	8 Ls-8 × 8 × 1	120.00
							2 Cov. Pls 54 × 1 1/4	: 135.00	2 Cov. Pls 54 × 1 1/8	121.50
	—Same as for Section 9—→						6 Ls-7 × 4 × 5/8	: 38.88	6 Ls-7 × 4 × 5/8	: 38.88
							1-Pl 34 × 3/4	: 25.50	1-Pl. 34 × 3/4	: 25.50
							2 Ls-4 × 4 × 1/2	: 7.50	2 Ls-4 × 4 × 1/2	: 7.50
							4 Ls-4 × 4 × 9/16	: 13.24	4 Ls-4 × 4 × 9/16	: 13.24
1		587.62		587.62		587.62		574.12		560.62
2		12,701		12,701		12,701		12,194		11,688
3		2,070		2,070		2,070		1,991		1,912
	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT	THRUST	MOMENT
4	+6,085	+917 +28	+6,018	+587 −254	+5,967	+151 −614	+5,931	−284 −1074	+5,911	−434 −1383
5	+535	+13,470	+618	+12,440	+699	+10,970	+740	+9,440	+729	+8,700
6	+803	−12,110	+716	−11,230	+678	−9,550	+677	−7,960	+731	−6,860
7	∓91	±2,212	∓91	±2,952	∓92	±3,505	∓93	±3,874	∓93	±4,058
8	∓23	±553	∓23	±738	∓23	±876	∓23	±968	∓23	±1,015
9	±208	±720	±94	±1,640	±20	±2,440	∓17	±4,150	∓17	±3,500
10										
11	+6,506	+17,152	+6,522	+16,767	+6,551	+15,502	+6,555	+13,998	+6,524	+13,339
12	+7,002	−14,847	+6,848	−15,174	+6,760	−14,545	+6,724	−13,876	+6,758	−13,316
13	+6,714	+17,872	+6,616	+18,407	+6,571	+17,942	+6,538	+18,148	+6,507	+16,839
14	+6,794	−15,567	+6,754	−16,814	+6,740	−16,185	+6,741	−18,026	+6,775	−16,816
15	FROM +M	FROM −M	FROM +M	FROM −M	FROM +M	FROM −M	FROM +M	FROM −M	FROM +M	FROM −M
16										
17	11.07	11.92	11.10	11.65	11.15	11.50	11.42	11.71	11.64	12.05
18	8.29	7.17	8.10	7.33	7.49	7.03	7.03	6.97	6.98	6.96
19	19.36	19.09	19.28	18.98	18.64	18.53	18.45	18.68	18.62	19.01
20										
21	11.43	11.56	11.26	11.49	11.18	11.47	11.39	11.74	11.61	12.08
22	8.63	7.52	8.89	8.12	8.66	8.21	9.12	9.05	8.81	8.79
23	20.06	19.08	20.15	19.61	19.84	19.68	20.51	20.79	20.42	20.87

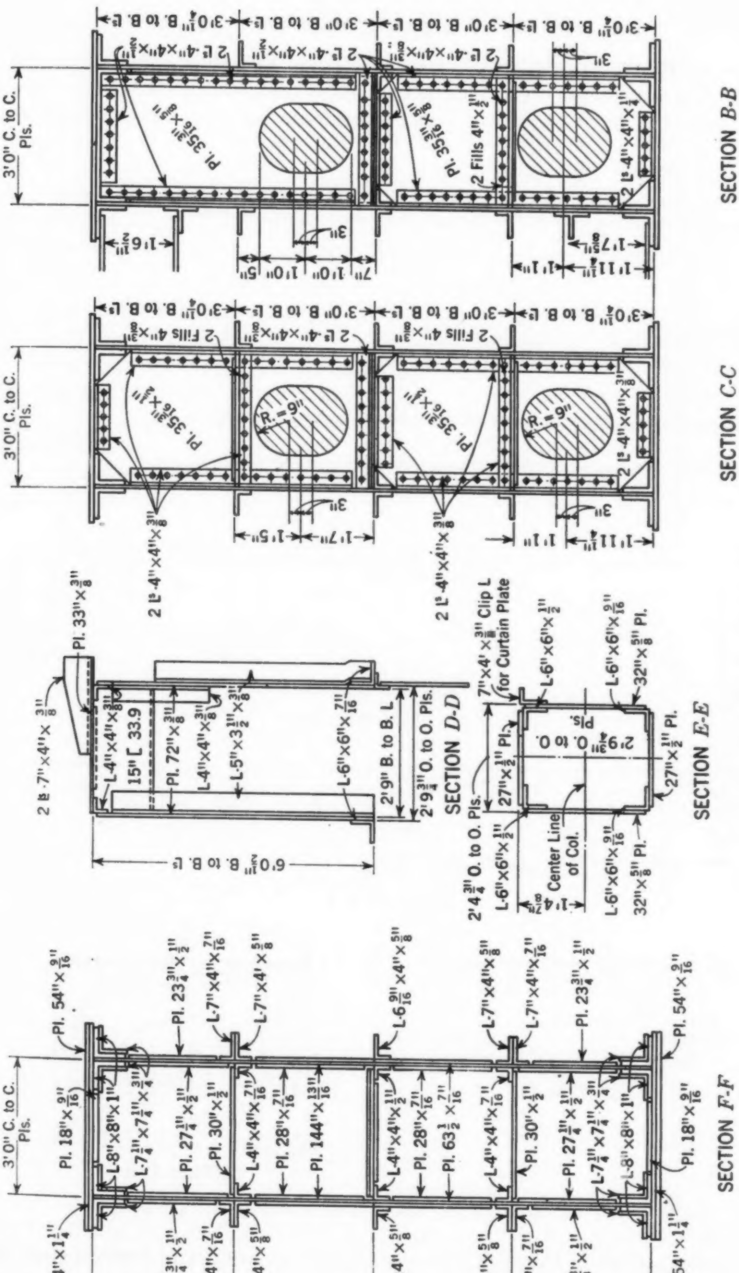


FIG. 8.—TYPICAL CROSS SECTIONS

ARCH ABUTMENTS

The abutments of each rib were constructed as independent units, and all four of the abutments were founded on solid rock which sloped up from the water at an angle of about 45°. All abutments were keyed into the rock by

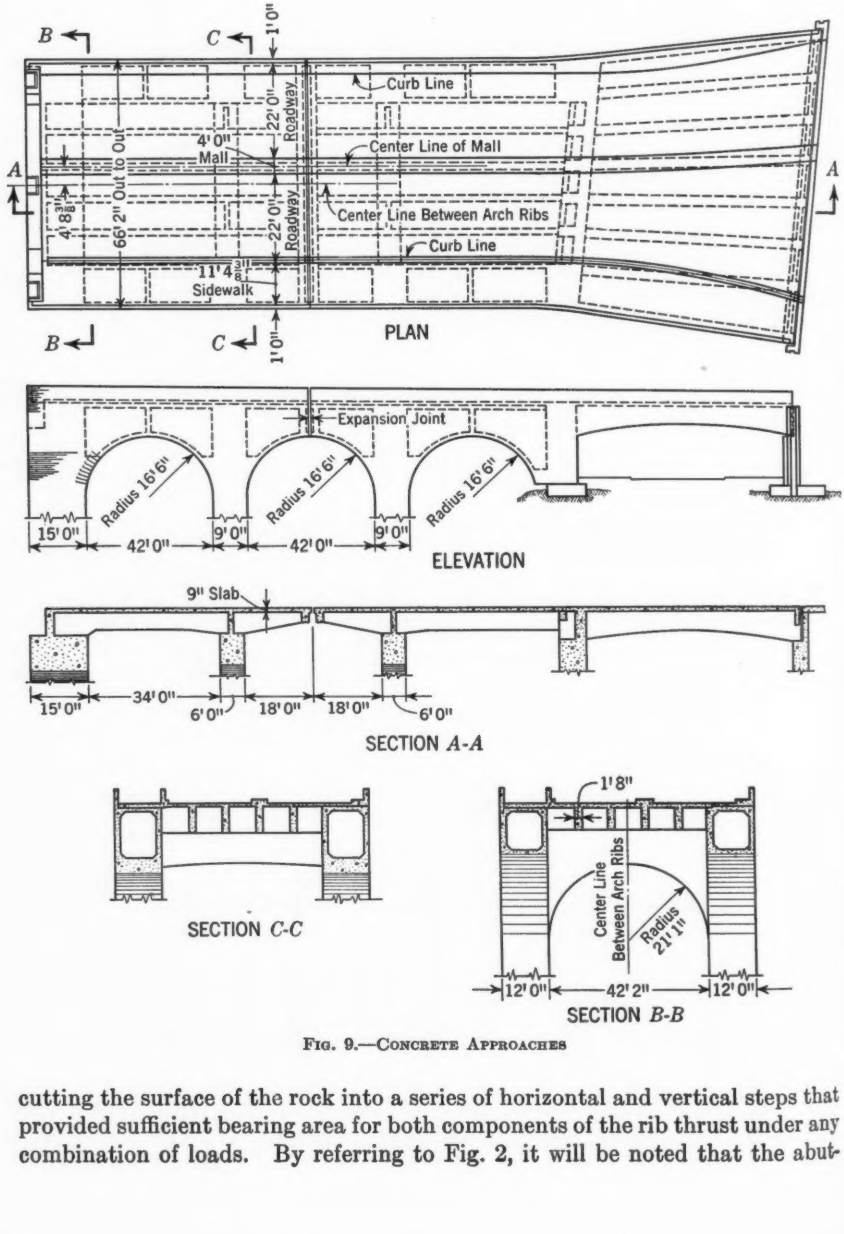


FIG. 9.—CONCRETE APPROACHES

cutting the surface of the rock into a series of horizontal and vertical steps that provided sufficient bearing area for both components of the rib thrust under any combination of loads. By referring to Fig. 2, it will be noted that the abut-

ments rest on the upper layers of the Albion sandstone, and exert thrust against the Irondequoit and Wolcott limestones and the Thorold sandstone.

Approaches

The reinforced concrete approach structures on the two sides of the river are similar. Each approach consists of three arched spans over the sloping walls of the gorge and a beam span over a transverse highway. The concrete box arches are 12 ft wide and frame into columns 12 ft in width. The bridge deck between the arches is carried by reinforced concrete beam and girder construction, as shown in the cross-sectional view in Fig. 9. The longitudinal view at the center line of the bridge indicates the method of framing for this part of the structure. An expansion joint at the crown of the middle arched span completely separates the two halves, so that they act as cantilevers. The design for the concrete structures was made to provide for continuity. The girders for the spread girder span over River Road on the Canadian side are particularly heavy, the north and south girders having span lengths of 85 ft and 91 ft, respectively.

Summary of Quantities

The principal quantities in the bridge are as follows:

Main Arch Span.—	
Concrete in deck, in cubic yards	1,800
Reinforcing bars in deck, in pounds	210,000
Reinforcing trusses for roadway slab, in pounds	300,000
Carbon steel in floor system, in pounds	2,410,000
Silicon steel in floor system (floor beams), in pounds	480,000
Carbon steel in spandrel columns, in pounds	1,140,000
Carbon steel in rib laterals, in pounds	900,000
Silicon steel in arch ribs and skewbacks, in pounds	6,220,000
Concrete in arch abutments, in cubic yards	7,100
Reinforcing steel in arch abutments, in pounds	300,000
American Approach.—	
Concrete, in cubic yards	7,800
Reinforcing steel, in pounds	840,000
Canadian Approach.—	
Concrete, in cubic yards	10,800
Masonry facing, in cubic feet	5,400
Reinforcing steel, in pounds	1,290,000

PART II.—DEFLECTION EFFECTS AND DESIGN FORMULAS FOR FLEXIBLE ARCHES

The elastic theory has been in use for many years in the design of structures. In general, any change of geometry of the structure due to strain is neglected in this theory. When the elastic displacements are small, the error involved is usually small; but, when the elastic displacements are appreciable, the error introduced may be an important factor in the design.

More exact methods have been developed for some of the problems of structural design in which deflections should be considered. The "deflection theory" for suspension bridges, which takes into account the elastic displacements of the structure, was first used in the United States in the design of the Manhattan Bridge in New York. It was amplified and developed further in the design of later suspension bridges. This theory, which results in lower bending moments in the stiffening truss of a suspension bridge than those calculated by the elastic theory, is considered applicable for practically all spans.

Columns under eccentric load or with initial curvature and columns under the combined action of axial and transverse loads continue to receive considerable attention. The rib of an arch bridge, being a compression member subject to both direct and bending stress, will have deflection characteristics similar to those of a column under the same type of loading.

Several deflection theories are available for arch ribs (see Appendix I). In each of these theories, certain assumptions have been made as to the variation of moment of inertia in the arch rib, and, in some cases, the part of span loaded with live load is specified. However, in one of these theories, referred to as "Stern's" in Appendix I, the arch axis can be divided into any desired number of sections, each having a constant moment of inertia, and live load can be placed on any part of the structure. This latter method was used to compute the quarter-point deflection for a preliminary two-hinged arch design, using the same loading as that required to produce the maximum elastic-theory moments at the quarter point. This computed deflection was 60% greater than the value given by the elastic theory.

When the Niagara design was started, a two-hinged arch rib was analyzed first by the elastic theory, with total stresses not exceeding those set up as allowable working stresses. The resulting rib, which was 14 ft in depth and weighed 3,000 lb per ft, then was cheked by an analysis which included the effect of the deflection. This more exact calculation showed that the quarter-point deflection of 12 in., due to (dead + live + impact)-load, as computed by the elastic theory, increased to 21 in. under the exact method, and that the true unit stresses were 28% greater than those indicated by the elastic theory. Perhaps more important, this calculation showed that an increase in dead and live loading of 30% above the design loads would increase the stresses to the yield strength of the steel, so that the safety factor of the structure, instead of being 1.875 as indicated by the elastic-theory unit stresses, was only 1.30. A two-hinged arch rib with a true safety factor of 1.875 under the design loads could have been obtained with a rib of 15-ft depth, weighing 3,800 lb per ft.

In the case of the two-hinged arch rib, the calculation of the quarter-point vertical deflection of 12 in. indicated that the rib had moved radially away from the thrust line approximately 13.6 in. This would produce an additional moment equal to the thrust at that point times the 13.6-in. displacement. In the case of this preliminary design, such a moment amounted to more than 8,600 ft-kips as compared to the elastic-theory moment of 22,950 ft-kips. Furthermore, this additional moment would produce still more deflection, which finally would total 23.4 in. in the radial direction, and a total additional moment of more than 14,500 ft-kips.

In contrast to the flexibility of the aforementioned two-hinged arch rib, the hingeless arch rib which was used in the final design of the bridge is 12 ft deep and weighs 2,600 lb per ft. It has a quarter-point deflection due to (dead + live + impact)-load of 5.5 in. by the elastic analysis. This increases to 6.9 in. by the more exact computation taking deflections into account.

The preliminary calculations for the two-hinged arch rib were sufficient to show that the elastic theory was inadequate for the design of such a structure. Had the two-hinged rib of the first preliminary design been built, the structure undoubtedly would have been unsatisfactory in its performance.

The results of the calculations also suggested the advisability of further study by the use of models, especially for those structures in which large displacements occur. Such a project was undertaken at Princeton University under the supervision of Professor Timby, as reported elsewhere in this Symposium. Three different models were used and a number of different loading conditions were applied to each.

The factor of safety (N), as used in the preceding paragraphs and elsewhere in this paper, is defined by the following relation:

$$N = \frac{W_{yp}}{W_w} \qquad (1)$$

in which W_{yp} is the (dead + live + impact)-load which, together with the effects of temperature and wind, produces a yield point stress in the arch rib, and W_w is the (dead + live + impact)-working load or design load. For this purpose W can be assumed to be the panel load.

The computations for the deflection effects in the preliminary two-hinged arch rib and for the hingless rib as built were based upon finding the equilibrium position of the arch rib by trial. That is, after computing the displacements from the elastic-theory moments and the change in moments resulting from the displaced position of the arch axis, correction moments to displace the arch rib to its equilibrium position were estimated. From these moments, displacements were calculated and second correction moments estimated; and this procedure was repeated until the equilibrium position of the rib had been determined. The values for deflections and moments computed by this procedure will be referred to herein as "exact."

Complete calculations were made for the loading condition that produces maximum bending moment at the quarter point of the span. After exact values were calculated and confirmed by the results of model tests, a simple, approximate procedure was developed which gives an accuracy sufficient for preliminary design and, in the ordinary structure, for final design.

Deflection Effects in Two-Hinged Arch Rib

The relations between the forces acting on the structure and the resulting radial displacements are expressed by the differential equation (see Fig. 10):

$$\frac{d^2u}{ds^2} + \frac{u}{(r_o)^2} = -\frac{M}{E\,I} - \frac{T}{r_o\,E\,A} \qquad (2)$$

in which ds = differential element of length of the arch rib; u = radial component of displacement of the rib; r_o = original radius of curvature of rib; M = final bending moments; T = final direct thrust; I = moment of inertia

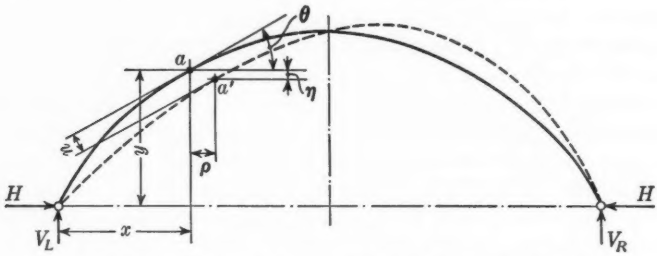


FIG. 10.—DEFLECTION EFFECT IN A TWO-HINGED ARCH RIB

of rib cross section; A = cross-sectional area of rib; and E = modulus of elasticity. The vertical and horizontal displacements η and ρ are

$$\eta = u \cos \theta \qquad (3a)$$

and

$$\rho = u \sin \theta \qquad (3b)$$

In the usual elastic analysis for the two-hinged arch, the condition for equilibrium at any point, A, is expressed by the equation,

$$M_{ae} = M'_a - H\, y_a \qquad (4)$$

If the effects of displacements are included, Eq. 4 becomes,

$$M_{af} = M'_a + \Delta M'_a - (H + \Delta H)\,(y_a - \eta_a) \qquad (5)$$

in which M'_a = simple beam moment at point, A; H = horizontal component of end reactions; y_a = vertical ordinate of point, A, nominal outline; η_a = vertical component of deflection of point, A; and $\Delta M'_a$ and ΔH = change in M'_a and H resulting from the change in geometry of the rib.

The correct position of the arch axis is one for which Eqs. 2 and 5 are satisfied. In the analysis, the terms $\frac{u}{(r_o)^2}$ and $\frac{T}{r_o\, E\, A}$ of Eq. 2 were neglected, since these terms are relatively small for ribs of the curvature considered. Eq. 2 then reduces to:

$$\frac{d^2u}{ds^2} = -\frac{M}{E\, I} \qquad (6)$$

As a first approximation, the moments M_e determined by the elastic analysis were applied to the rib and the resulting displacements determined. If the rib is considered to be in this deformed position, it is not in equilibrium under the action of the computed forces because Eq. 5 is not satisfied.

A second approximation was made by calculating the moments M_1 for the rib in the deformed position, the coordinates being $x + \rho$ and $y - \eta$, instead of x and y.

The moments M_1 differed from the moments M_e by $\Delta m_1 = M_1 - M_e$. This difference for the two-hinged arch can be considered to consist of the following terms:

$$\Delta m_1 = \Delta M' - y\,\Delta H + H\,\eta + \eta\,\Delta H \qquad (7)$$

By applying these moments to the rib and finding the resulting deflections, moments M_2 can be computed from the new deformed position. By continuing this procedure, the equilibrium position of the arch when the moments and deflections are consistent can be determined.

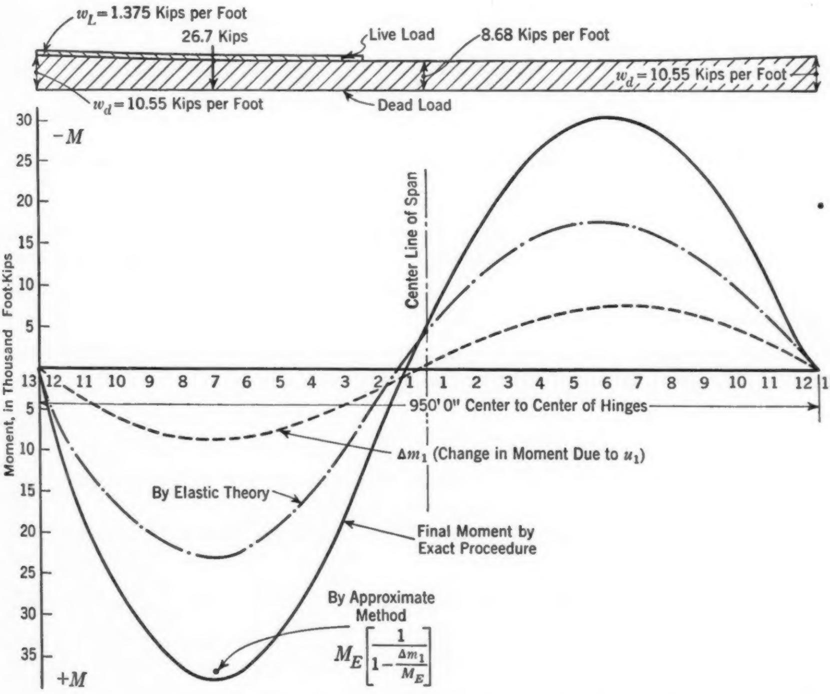


FIG. 11.—VARIATION IN M_e FOR A TWO-HINGED ARCH BRIDGE (L = 950 FT; RISE = 150 FT; I = 116.14 FT⁴ AT THE CROWN AND 120.24 FT⁴ AT THE QUARTER POINT)

For the Niagara two-hinged arch rib, it was noted that the variation in Δm_1, under loadings which produce maximum moment in sections near the quarter points of the span, was similar to the variation in M_e (Fig. 11), and that a close approximation could be obtained by assuming that the deflections at point A in the vicinity of the quarter point, produced by moment Δm_1, are given by:

$$\Delta u_{a1} = u_{ae}\left(\frac{\Delta m_{a1}}{M_{ae}}\right) \qquad (8)$$

Similarly the additional moment Δm_{a2}, resulting from the deflection Δu_{a1}, is given by:

$$\Delta m_{a2} = \Delta m_{a1}\left(\frac{\Delta u_{a1}}{u_{ae}}\right) = \Delta m_{a1}\left(\frac{\Delta m_{a1}}{M_{ae}}\right) = M_{ae}\left(\frac{\Delta m_{a1}}{M_{ae}}\right)^2 \ldots\ldots (9)$$

Assuming successive corrections to be obtained in a similar manner, the final moment M_{af} at point A can be written:

$$M_{af} = M_{ae} + M_{ae}\left(\frac{\Delta m_{a1}}{M_{ae}}\right) + M_{ae}\left(\frac{\Delta m_{a1}}{M_{ae}}\right)^2 + M_{ae}\left(\frac{\Delta m_{a1}}{M_{ae}}\right)^3 + \cdots (10)$$

or

$$M_{af} = M_{ae}\left(\frac{1}{1 - \dfrac{\Delta m_{a1}}{M_{ae}}}\right) \ldots\ldots (11)$$

An approximate value for the radial displacement can be obtained in a similar manner, giving:

$$u_{af} = u_{ae} + u_{ae}\left(\frac{\Delta m_{a1}}{M_{ae}}\right) + u_{ae}\left(\frac{\Delta m_{a1}}{M_{ae}}\right)^2 + \cdots = \frac{u_{ae}}{1 - \dfrac{\Delta m_{a1}}{M_{ae}}} \ldots (12)$$

Furthermore, it was found that, for a rib of the Niagara arch properties, the following relation was a very close approximation:

$$\Delta m_{a1} = T_a\, u_{ae} \ldots\ldots (13)$$

in which T_a = the total axial thrust at the point A and u_{ae} = the radial displacement of point A as computed by the elastic theory.

Substituting Eq. 13 in Eqs. 9 and 10, the following expressions result:

$$M_{af} = M_{ae}\left[\frac{1}{1 - T_a\left(\dfrac{u_{ae}}{M_{ae}}\right)}\right] \ldots\ldots (14a)$$

and

$$u_{af} = u_{ae}\left[\frac{1}{1 - T_a\left(\dfrac{u_{ae}}{M_{ae}}\right)}\right] \ldots\ldots (14b)$$

A comparison of the moments computed by Eq. 14*a* and those obtained by model tests at Princeton University (described in the succeeding paper in this Symposium) shows that the agreement is quite good when the value of the term in brackets in Eqs. 14 is less than 2.

For a perfectly fabricated rib, the final (dead load + live load + impact load) moment being given by Eq. 14*a*, the total resulting (dead load + live load + impact load) stress in the extreme fiber at a point A (this point being

assumed as between 0.10 L and 0.30 L from either hinge) can be written as:

$$s = \frac{T_a}{A_a} \pm \frac{M_{ae}}{S_a}\left[\frac{1}{1 - T_a\left(\frac{u_{ae}}{M_{ae}}\right)}\right] \qquad (15)$$

in which A_a = cross-sectional area of rib at the point A; and S_a = section modulus of rib at the point A.

If the useful capacity of the structure is defined as the load that produces a stress at any section equal to the elastic limit s_y of the material and the factor of safety N, as previously defined, is introduced, the following equation should be satisfied by a properly designed rib:

$$s_y = N\left(\frac{T_a}{A_a}\right) + N\,\frac{M_{LL+I}}{S_a}\left[\frac{1}{1 - N\,T_a\left(\frac{M_{ae}}{M_{LL+I}}\right)}\right] + \frac{N\,M_{DL}}{S_a} + \frac{T_t + T_w}{A_a} + \frac{M_t + M_w}{S_a} \qquad (16)$$

in which T_a is the elastic-theory thrust at point A due to (dead + live + impact)-load; T_t and T_w, the thrusts at that point due to temperature effect and wind, respectively; and M_{DL}, M_{LL+I}, M_t, and M_w are the elastic-theory moments due to (dead + live + impact)-load, temperature effect, and wind, respectively.

In the special case of a perfectly formed rib carrying loads that produce no bending moment, the following condition should also be satisfied: $N\,T_{0.25}\left(\frac{u_{0.25E}}{M_{0.25E}}\right) < 1$—in which $u_{0.25E}$ and $M_{0.25E}$ are radial displacement and the moment that would be produced at the quarter point of the span by an assumed uniform live load over half the span, and $T_{0.25}$ is the thrust at the quarter point due to dead load only. Inspection of Eq. 14*b* shows that as the term $T_{0.25}\,\frac{u_{0.25E}}{M_{0.25E}}$ approaches unity, any small displacement of the rib at the one-quarter point becomes very large. The rib, therefore, would buckle in a two-loop form. The value of thrust which makes this term equal to unity agrees closely with the critical buckling thrust obtained for special cases by more nearly mathematically exact theories.[4]

Practically, of course, it is impossible to fabricate and erect an arch rib that will conform exactly to a prescribed outline. The worst form of the variation for accidental errors in rib outline would be the same variation as the live load displacements, u. With accurate workmanship, the maximum probable ordinate of a departure of this type can be expressed approximately as: $e' = \frac{\sqrt{L}}{30}$, but not less than 0.25 . . . when e' is in inches and L, in feet.

For the Niagara arch span, L = 950 ft, e' = 1.03 in.; and, for a span of 225 ft, the value of e' would be 0.5 in.

[4] "Theory of Elastic Stability," by S. Timoshenko, 1st Ed., 1936, p. 225.

An error of this magnitude, although insignificant in an elastic-theory analysis, may produce appreciable secondary moments and should be considered in design. The effect of such error incorporated in Eq. 16 gives finally:

$$\frac{s_y}{N} = \frac{T_a}{A} + \frac{M_{LL+I} + T_a \frac{e'}{N}}{S_a} \times \frac{1}{1 - N\, T_a \frac{u_{ae}}{M_{LL+I}}}$$

$$+ \frac{M_{DL}}{S_a} + \frac{1}{N}\left(\frac{T_t + T_w}{A_a} + \frac{M_t + M_w}{S_a}\right) \ldots\ldots\ldots\ldots (17)$$

Eq. 17 is proposed for design use. The following values of s_y and N, which have been taken from the 1941 A.R.E.A. Specifications for Steel Railway Bridges, Appendix A, indicate the numerical values used in current design for (dead + live + impact)-load stresses:

	s_y (lb per sq in.)	N	$\frac{s_y}{N}$
Structural steel	33,000	1.833	18,000
Silicon steel	45,000	1.875	24,000
Nickel steel	55,000	1.83	30,000

When unusual outline or loadings are encountered, or when the importance of the structure warrants, a check of Eq. 16 can be obtained at a few critical points by successive approximations of the elastic theory, previously described and referred to as the "exact method."

As a numerical example, Eq. 15 will be applied to a preliminary two-hinged arch design for the Niagara span: The stresses at the quarter point are shown in Table 2. For this case: The depth of the rib is 14 ft; the section area is 614

TABLE 2.—STRESSES AT THE QUARTER POINT IN A PRELIMINARY TWO-HINGED ARCH

Loading	Elastic Theory				Deflection Theory	
	Thrust (Kips)		Moment (Ft-Kips)		Thrust (Kips)	Moment (Ft-Kips)
	$+M$	$-M$	$+M$	$-M$	$+M$	$+M$
Dead load	−7,126	−7,126	+353	+353	−7,554	353
Live load	−425	−709	+22,594	−22,076		+37,123
Impact	−37	−62	+1,990	−1,943	−37	+3,270
Σ	−7,588	−7,897	+24,937	−23,666	−7,591	+40,746

sq in.; the section modulus is 29,000 in.³; u_{ae} = 13.6 in. for positive moment and 14.0 in. for negative moment. Using the moments and thrusts computed by the elastic theory, the following stresses are found:

For maximum positive moment—

$$s = \frac{7{,}588}{614} + \frac{24{,}937 \times 12}{29{,}000} = 12.36 + 10.31 = 22.67 \text{ kips per sq in.}$$

For maximum negative moment—

$$s = \frac{7,897}{614} + \frac{23,666 \times 12}{29,000} = 12.86 + \ \ 9.79 = 22.65 \text{ kips per sq in.}$$

The stress, computed from the moments and thrusts obtained by the deflection analysis, is:
For maximum positive moment—

$$s = \frac{7,591}{614} + \frac{40,746 \times 12}{29,000} = 12.36 + 16.86 = 29.22 \text{ kips per sq in.}$$

Using the approximate Eq. 15, the resulting stresses are:
For maximum positive moment—

$$s = \frac{7,588}{614} + \frac{24,584 \times 12}{29,000} \times \frac{1}{1 - 7,588 \times \dfrac{13.6}{12 \times 24,584}}$$

$$+ \frac{353 \times 12}{29,000} = 12.36 + 10.17 \times \frac{1}{0.6502} + 0.15 = 28.15 \text{ kips per sq in.}$$

For maximum negative moment—

$$s = \frac{7,897}{614} + \frac{24,019 \times 12}{29,000} \times \frac{1}{1 - 7,897 \times \dfrac{14.0}{12 \times 24,019}}$$

$$- \frac{353 \times 12}{29,000} = 12.86 + 9.94 \times \frac{1}{0.6164} - 0.15 = 28.83 \text{ kips per sq in.}$$

Deflection Effect in a Hingeless Arch Rib

In the elastic analysis for the fixed arch rib, the condition for equilibrium at any point A is expressed by the equation:

$$M_a = M'_a + M_{oa} - H\, y_a \qquad (18)$$

When the effects of displacements are included, Eq. 18 becomes:

$$M_{af} = M'_a + \Delta M'_a + M_{oa} + \Delta M_{oa} - (H + \Delta H)\,(y_a - \eta) \qquad (19)$$

in which M_{oa} = moment at point A due to end moments; and $\Delta M'_a$, ΔH, ΔM_{oa} = increments resulting from the change in geometry of the rib. The correct position of the arch axis is that for which Eqs. 2 and 19 are satisfied.

The equilibrium position was determined for the Niagara arch rib for the live-load condition which produces maximum positive moment at the quarter point of the span. This was accomplished in a manner similar to that used for the two-hinged rib. The results of this analysis are shown partly in Fig. 12 in which the first correction moment Δm_1 is approximately proportional to the moments of the elastic theory. Assuming then that this relation exists, the final moment and radial displacement can be estimated by the same procedure used in obtaining Eqs. 10 and 12 for the two-hinged arch.

For the hingeless arch, these equations cannot be simplified by introducing $T\, u_e$ as an approximation for Δm_1, as in the case of the two-hinged arch.

The stress is given by:

$$s = \frac{T_a}{A_a} \pm \frac{M_{ae}}{S_a}\left(\frac{1}{1 - \frac{\Delta m_{a1}}{M_{ae}}}\right) \qquad (20)$$

The moment produced by an initial error in rib outline was found to be about 70% of the moment that would be produced in a two-hinged rib by the same

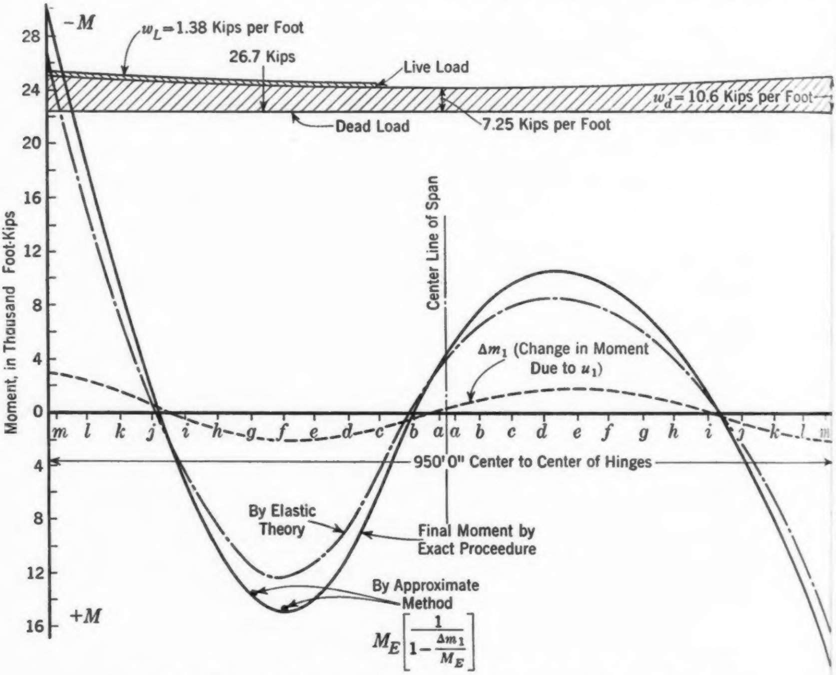


FIG. 12.—VARIATION IN M_e FOR A FIXED ARCH (L = 950 FT; RISE = 150 FT; I = 81.17 FT^4 AT THE CROWN AND 88.20 FT^4 AT THE QUARTER POINT)

error. Therefore a properly designed hingeless arch rib should satisfy the following equation:

$$\frac{s_y}{N} = \frac{T_a}{A_a} + \frac{M_{LL+I} + 0.7\, T_a \frac{e'}{N}}{S_a}\left[\frac{1}{1 - N\left(\frac{\Delta m_{a1}}{M_{LL+I}}\right)}\right] + \frac{M_{DL}}{S_a} + \frac{1}{N}\left[\frac{T_t + T_w}{A_a} + \frac{M_t + M_w}{S_a}\right] \qquad (21)$$

The calculations for Δm_{a1} are quite lengthy as the rib outline is generally unsymmetrical after the first deflections are added to the ordinates. However, when a sufficient variety of ribs have been computed exactly, or when model tests, comparable to those made at Princeton University for the two-hinged rib, have been made for the hingeless rib, it may be possible to write an approximate expression for the value Δm_1.

As a numerical example Eq. 20 will be applied to the final Niagara ribs. Similarly to Table 2, the stresses at the quarter point are shown in Table 3. For this case: The depth of rib is 12 ft; the section area, 553.96 sq in.; the section modulus, 24,600 in.^{3} and $\Delta m_{a1} = 2{,}018$ ft-kips, for positive moment, and $\Delta m_{a1} = -1{,}888$ ft-kips for negative moment.

TABLE 3.—STRESSES AT THE QUARTER POINT IN THE HINGELESS NIAGARA ARCH

Loading	Elastic Theory				Deflection Theory	
	Thrust (Kips)		Moment (Ft-Kips)		Thrust (Kips)	Moment (Ft-Kips)
	$+M$	$-M$	$+M$	$-M$	$+M$	$+M$
Dead load	−6,062	−6,105	+2,138	+657	−6,455	+16,628
Live load	−398	−763	+11,948	−10,640		
Impact	−35	−67	+1,051	−937	−35	+1,230
Σ	−6,495	−6,935	+15,137	−10,920	−6,490	+17,858

As before, using the moments and thrusts computed by the elastic theory, the following stresses are found:

For maximum positive moment—

$$s = \frac{6{,}495}{553.96} + \frac{15{,}137 \times 12}{24{,}600} = 11.72 + 7.38 = 19.10 \text{ kips per sq in.}$$

For maximum negative moment—

$$s = \frac{6{,}935}{553.96} + \frac{10{,}920 \times 12}{24{,}600} = 12.52 + 5.33 = 17.85 \text{ kips per sq in.}$$

The stress obtained from the deflection-theory moments and thrusts is as follows:

For maximum positive moment—

$$s = \frac{6{,}490}{553.96} + \frac{17{,}858 \times 12}{24{,}600} = 11.72 + 8.71 = 20.43 \text{ kips per sq in.}$$

Using the approximate Eq. 20, the following stresses are obtained:

For maximum positive moment—

$$s = \frac{6{,}495}{553.96} + \frac{12{,}999 \times 12}{24{,}600} \times \frac{1}{1 - \dfrac{2{,}018}{12{,}999}} + \frac{2{,}138 \times 12}{24{,}600}$$

$$= 11.72 + 6.34 \times \frac{1}{0.8448} + 1.04 = 20.27 \text{ kips per sq in.}$$

For maximum negative moment—

$$s = \frac{6{,}935}{553.96} + \frac{11{,}577 \times 12}{24{,}600} \times \frac{1}{1 - \dfrac{1{,}888}{11{,}577}} - \frac{657 \times 12}{24{,}600}$$

$$= 12.52 + 5.65 \times \frac{1}{0.8369} - 0.32 = 18.95 \text{ kips per sq in.}$$

Conclusions

Four basic conclusions seem justifiable from this paper:

1. The useful strength of an arch rib is dependent in part upon the magnitude of the product of deflection under live load and total thrust.

2. In flexible arches the usual method of applying the elastic theory will lead to an appreciable overestimate of the strength of the structure. This error is especially serious in the case of long spans with heavy dead load, such as the Niagara arch bridge.

3. An attempt has been made to set up practical design formulas, for both two-hinged and hingeless ribs, that include deflection effects.

4. Additional model tests would aid greatly in confirming and simplifying the basis of design suggested in this article.

APPENDIX I

Résumé of Available Deflection Theories

J. Melan [5] developed a deflection theory for two-hinged arch ribs with parabolic axis and constant cross section. Horizontal displacement components were neglected and the further approximation was used that $\frac{d^2u}{ds^2} = \frac{d^2\eta}{dx^2}$.

S. Kasarnowsky [6] obtained a solution for the deflection stresses in a constant-section two-hinged arch rib with parabolic arch axis. He assumed that $\eta = u \cos \theta$ and that there was no horizontal displacement. The analysis is restricted to the case of live load extending over one half of the span.

B. Fritz [7] developed equations for three-hinged, two-hinged, and hingeless arch ribs of constant cross section and parabolic arch axes. Horizontal displacements were neglected and the assumption was made that $\eta = u \cos \theta$. Fritz's equations are applicable for a uniform live load over any part of the span length.

A. Freudenthal [8] assumed that the radial displacement is equal to the vertical component of deflection η. He obtained a solution for the stresses in either a two-hinged rib or a hingeless arch rib with parabolic axis and a variation of cross section given by $I = I_c \sec \theta$.

In a deflection theory for two-hinged arch ribs, George Stern,[9] Assoc. M. Am. Soc. C. E., assumed that there were no horizontal displacements of vertical loads and that $\eta = u \cos \theta$. The variation in cross section of rib was taken into account by assuming the arch to be divided into any number of subdivisions with constant section over each element. This theory may be used for any shape of arch axis and any loading condition.

[5] "Genauere Theorie des Zweigelenkbogens mit Beruecksichtigung der durch die Belastung erzeugten Formaenderung," by J. Melan, *Handbuch der Ingenieur-wissenschaften*, Vol. 2, 1906.

[6] "Beitrag zur Theorie weitgespannter Brueckenbogen mit Kaempfergelenken," by S. Kasarnowsky, *Stahlbau*, Copy 6, 1931.

[7] "Theorie und Berechnung vollwandiger Bogentraeger bei Beruecksichtigung des Einflusses der Systemverformung," by B. Fritz, Berlin, 1934.

[8] "Deflection Theory for Arches," by A. Freudenthal, publications by International Association for Bridge and Structural Engineering, Vol. 3, 1935.

[9] "A Deflection Theory of Two-Hinged Arch Ribs," by G. Stern, thesis No. 516, Dept. of Civ. Eng., Columbia Univ., New York, N. Y. (a résumé of theoretical studies made at the Technical Univ. of Vienna, Austria).

MOMENTS IN FLEXIBLE TWO-HINGED ARCHES

BY ELMER K. TIMBY,[10] ASSOC. M. AM. SOC. C. E., LAWRENCE M. MEAD, JR.,[11] JUN. AM. SOC. C. E., AND ROBERT MCELDOWNEY, JR.,[12] ESQ.

SYNOPSIS

The research described in this paper was planned to furnish information to assist in the design of arch ribs sufficiently flexible to sustain appreciable deflections before becoming overstressed. Experience has shown that the classic elastic theory, when applied to such arches, yields calculated stresses which are considerably less than existing stresses unless extended by successive approximations which consider the effect of the rib deflections on the moments. The work consisted of experimental observations of the behavior of loaded circular two-hinged steel arch models of constant section. The results of these observations are presented graphically and have been treated analytically to provide: (*a*) Practical limits of flexibility for the design of two-hinged arch ribs by straight elastic theory; (*b*) material assistance in the preliminary design of arches; and (*c*) actual design methods for certain two-hinged arch ribs which are too flexible for analysis by elastic theory. Within reasonable limits, the results may be applied to noncircular profiles and to varying sections by use of proper relative dimensions.

TEST PROCEDURE

Experimental observations were made on three models, having properties described in Fig. 13. Each rib had a section 1.00 in. wide by 0.25 in. deep and was formed cold to fit a template of the desired circular radius to within 0.01 in. Material was cold finished steel with an average value for E of 29,520,000 lb per sq in., maximum variation noted being 1.03%. One of these models is shown mounted in the special loading frame built for these tests in Fig. 14.

Both dead load and live load, accurate to 0.01 lb, were suspended from the model on hanger rods (Fig. 14) at the E-points (Fig. 13). Several different values of dead load, $\overline{DL}$, were used, and each was defined by its intensity in pounds per unit of length at the crown. The intensity of any particular dead load was varied along the arch in a manner calculated to keep the resultant dead-load thrust line on the center line of the rib. Live loads were maintained uniform for the extent of the loading.

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Dead loads and live loads designed and used on the models are listed in Table 4, in which R denotes the ratio of dead load to live load ($\overline{DL}/\overline{LL}$). These various loads were designed to extend beyond practical limits in both directions of magnitude, varying from loads which would be considered very light, in view of the stiffness of the section, to loads which caused the arch to approach a buckling condition. The particular loads that corresponded most closely to the original two-hinged design for the Niagara Rainbow arch mentioned in the preceding paper in the Symposium were: $\overline{DL} = 2.50$ lb per in.; $\overline{LL} = 0.50$ lb per in.; $R = 5.00$.

TABLE 4.—LIVE LOADS, IN POUNDS PER INCH

Dead-load intensity, $\overline{DL}$, in lb per in.	Values of $R\left(=\frac{\overline{DL}}{\overline{LL}}\right)$				
	2.50	5.00	7.50	10.00	15.00
1.25	0.50	0.25	0.167	0.125	0.083
2.50	1.00	0.50	0.333	0.250	0.167
3.75	1.50	0.75	0.500	0.375	0.250
5.00	2.00	1.00	0.667	0.500	0.333

The models were mounted in the apparatus shown in Fig. 14, and observations were made to determine deflection, horizontal component at the reactions, and maximum moment at a point (only the latter being presented in this paper). All observations for maximum moment were made between the condition of

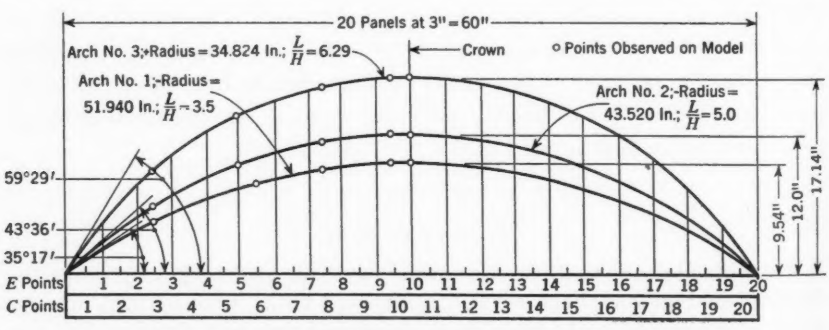

FIG. 13.—CONTROLLING DIMENSIONS OF MODEL ARCH RIBS TESTED

dead load on the arch, and the condition of dead load plus live load placed to produce the maximum value of moment at the point being studied—the extent and position of the live load being determined by trial. The hinges were made free to rotate by using clean ball bearings and vibrators at each reaction. Checks indicated that friction was negligible. A constant span was maintained by a horizontal adjustment of one support during each loading cycle to provide a specified constant distance between hinges. This distance was controlled by two 0.0001-in. micrometer dials, shown at the left reaction in Fig. 14.

Deflection measurements were made in two ways: (1) By direct observation with filar micrometer microscopes; and (2) by observing with a microscope a photographic film on which exposures had been made for both the unloaded and the loaded condition, in respect to live load. The latter method checked

the first for accuracy and proved to be much more rapid, in addition to preserving the information for observation and study at a later date. Fig. 15 indicates such a double exposure for one of the more flexible conditions. Although photographs were made for each condition of loading, time has not

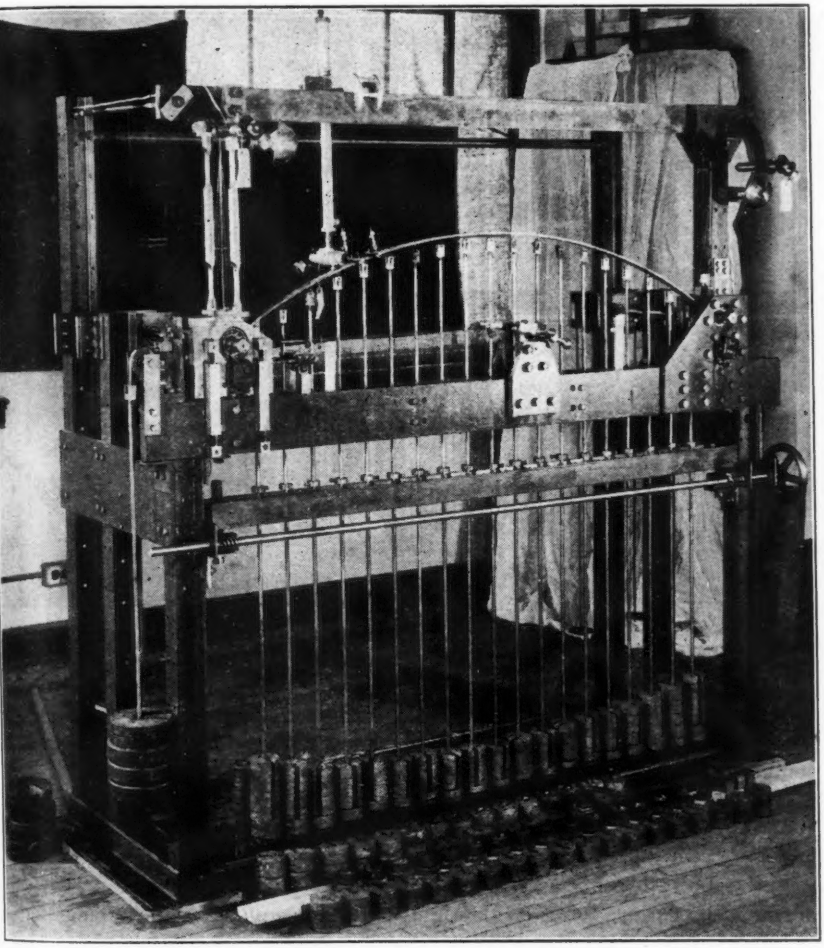

FIG. 14.—VIEW OF MODEL IN SPECIAL LOADING FRAME

permitted the study of these data, except in certain instances when they were used in conjunction with the observed horizontal reaction component and the statically determined vertical components, to check the directly observed moments.

The horizontal component was measured as follows: One hinge was mounted in a block fixed in the loading frame, whereas the other hinge was mounted

elastically in a block relatively free to translate in the direction of the span length. Calibrated piano wires, long enough to allow the use of 20-in. extensometers, with one end attached to the movable block and the other attached to blocks capable of being moved by worm threads and gears, were used

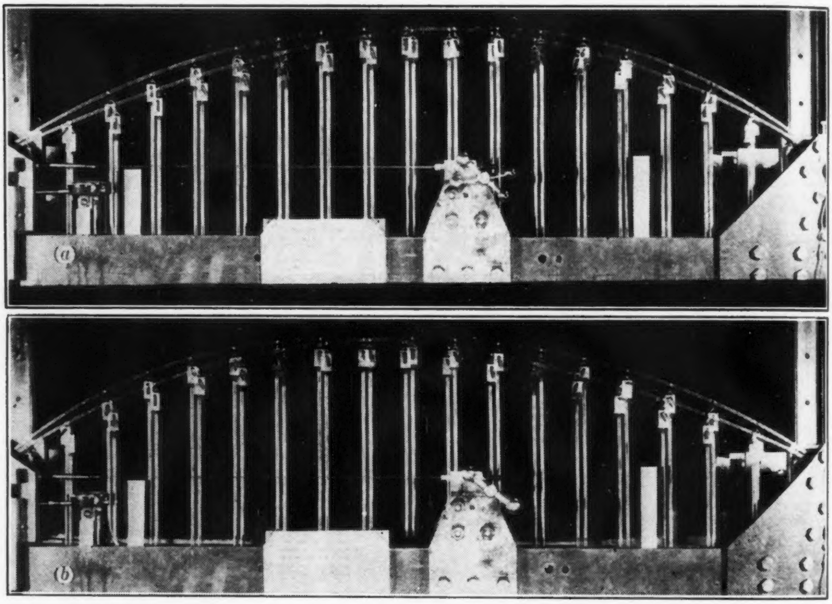

FIG. 15.—DOUBLE-EXPOSURE PHOTOGRAPHS TO OBSERVE DEFLECTIONS; MODEL OF ARCH NO. 1 AT POINT 6 C

to adjust the movable block to correct position for accurate span length. The strain readings on the extensometers indicated the magnitude of the horizontal component.

Moment in the arch rib was determined by

$$M = E\,I\,\frac{\Delta\theta}{\Delta L} \qquad (22)$$

in which the various terms on the right side of the equality sign were evaluated as follows:

$E\,I$—by calipered dimensions and repeated stress-strain measurements on a piece of steel cut from that used in making the model.

$\Delta\theta$—by use of two optically flat mirrors attached to the two ends of the elastic element (see Fig. 13 at point 5 E). In the plane of the model and about 15 ft from its near end, $\frac{1}{20}$-in. scales were supported on an engineer's transit. The images of these scales were observed in the mir-

rors through the transit, and the readings translated into differential angular deflections.

ΔL—by a mechanical spacing bar used in setting mirror mountings in position. The value used was 2.00 in. measured along the rib.

The accuracy of the equipment and the reliability of the experimental technique were such that reported results should be accurate within an error of not more than 3%.

The apparatus was equipped also with levers and weight pans designed to introduce, at the reactions, moments sufficient to return the springing planes to their normal positions, thus producing fixed-end conditions and the means of measuring the fixed-end moments. To enable the apparatus to be used for the study of flexible fixed-end arches, levers and weight pans were provided, designed to introduce, at the hinged reactions, moments sufficient to maintain the springing planes in their normal position. This produced fixed-end conditions and a means of measuring the fixed-end moments. Time permitted only a few tests of fixed-end conditions, but these indicated that further study would be profitable.

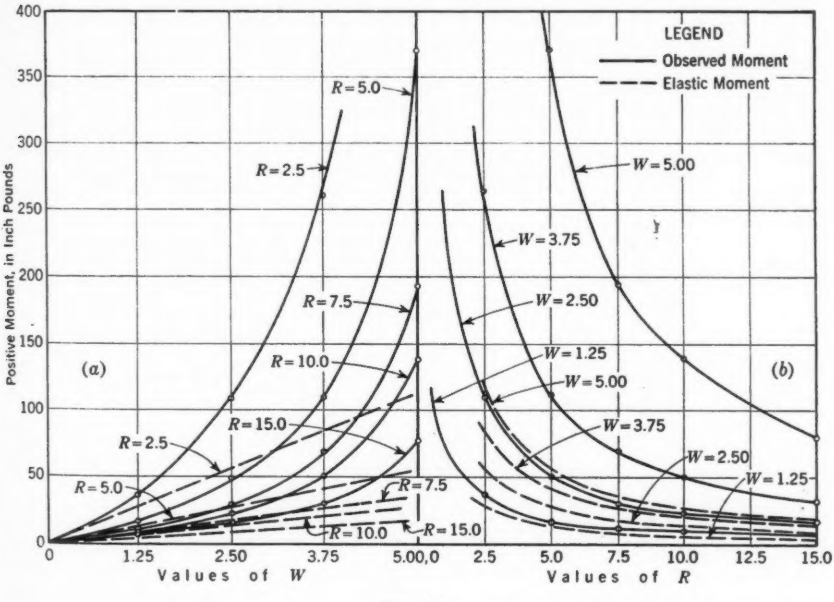

FIG. 16

EXPERIMENTAL DATA

Maximum positive and negative moments were observed at five different points in the semispan of each of the three arches tested and for the complete range of loadings in Table 4. Fig. 16 shows two typical linear plots of the same data for maximum positive moment M at point 6 C of arch No. 1, and

indicates the consistency of the observed data. Values of the moment as given by the elastic theory also are shown on these figures for comparison. In most cases the differences are rather marked.

Fig. 17 is a log-log plot of the same data as in Fig. 16, using the same coordinates but omitting the elastic-theory data and including the maximum

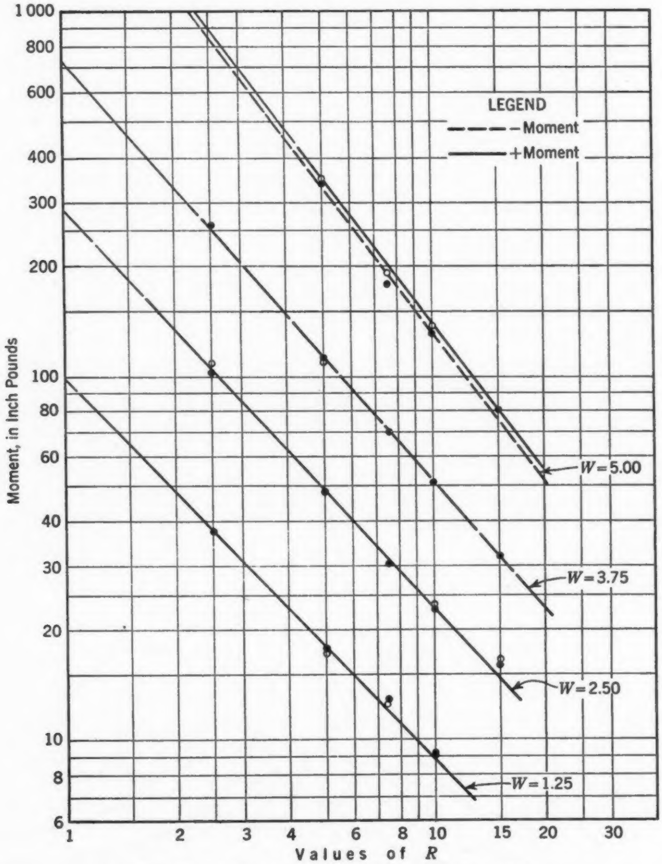

FIG. 17.—LOG-LOG PLOT OF DATA IN FIG. 16(b)

negative moment ($-M$) data. It will be observed that straight lines approximate the trends with remarkable accuracy. This tendency was found to be general for all the points on all arches within certain ranges whose extremes have been adopted as the limits of practicality for the use of the data presented in this paper.

These data are more useful to practical engineers if presented in terms of arches whose dimensions correspond more closely to those of actual structures. For this reason, the complete data for maximum moments were translated to refer to so-called "base arches," of similar geometric shape (in elevation)

to that of the models. In these base arches, $L = 1{,}000$ ft; $I = 100$ ft^4; $E = 29{,}000{,}000$ lb per sq in.; and loads and other values are proportional to those of the models. This translation was accomplished by applying to the model results the laws of similitude, familiar to any engineer using these data, in which

$$\overline{DL}_b = \overline{DL}\,\frac{E_b\,I_b}{E\;\,I}\left(\frac{L}{L_b}\right)^3 \qquad (23a)$$

and

$$M_b = M\,\frac{E_b\,I_b\,L}{E\;\,I\;\,L_b} \qquad (23b)$$

Values without a subscript in Eqs. 23 refer to any arch such as the model, and the values with the subscript, b, refer to the base arch. All subsequent data presented in this paper refer to these base arches.

Space prevents including a complete set of curves for the base arches similar to those of Fig. 17. These curves, complete sets of curves of the type shown subsequently in Figs. 19 and 20, and additional data not presented herein are available in the form of a thesis entitled "Moments in Flexible Arches," by Messrs. McEldowney and Mead, presented to the School of Engineering, Princeton University, in partial fulfilment of the requirements for the degree of Master of Science.

Equations for Maximum Moment.—The fact that the observed data plot as straight lines on log-log paper (see Fig. 17) suggested the possibility of writing equations for these lines by the usual intercept-and-slope relation. Calculations for the lines of Fig. 17 show that their slopes, $\frac{\log M}{\log R}$, also vary in a linear manner with the value of log M for $R = 1$ (the intercept of Fig. 17). Furthermore the values of log M for $R = 1$, the intercept, are found to vary approximately linearly with $\overline{DL}$. It was possible therefore to write two equations for the intercepts and slopes of the lines of Fig. 17 in terms of $\overline{DL}$ and log M for $R = 1$, respectively. These two equations then could be substituted into the slope-intercept equation for the lines of Fig. 17, giving a single equation for all the lines for M in terms of $\overline{DL}$, R, and four empirical coefficients, C_1, C_2, C_3, and C_4. This equation is general for all points on each of the three arches and, for the base arches, reduces to

$$\log_{10} M_b = (C_1\,\overline{DL}_b + C_2)\,(1 - C_3 \log_{10} R) - C_4 \log_{10} R \qquad (24)$$

in which C_1, C_2, C_3, and C_4 are selected for any particular point and any arch from the curves of Fig. 18.

By use of Eq. 24 and its accompanying coefficient curves it is possible to obtain values of the maximum bending moment, M_b: For any point on the three arches; for any selected value of $\overline{DL}_b$; and for R within the range of the tests. It should be emphasized that the position of the loading is not needed by the designer, since this was determined experimentally when testing. A designer need know merely $E\,I$, $\overline{DL}$, $\overline{LL}$ and the ratio of rise to span. Having converted these to the base arch, the designer can enter Fig. 18 at the selected span point and pick the coefficients for use in Eq. 24. The calculations are rapid and can

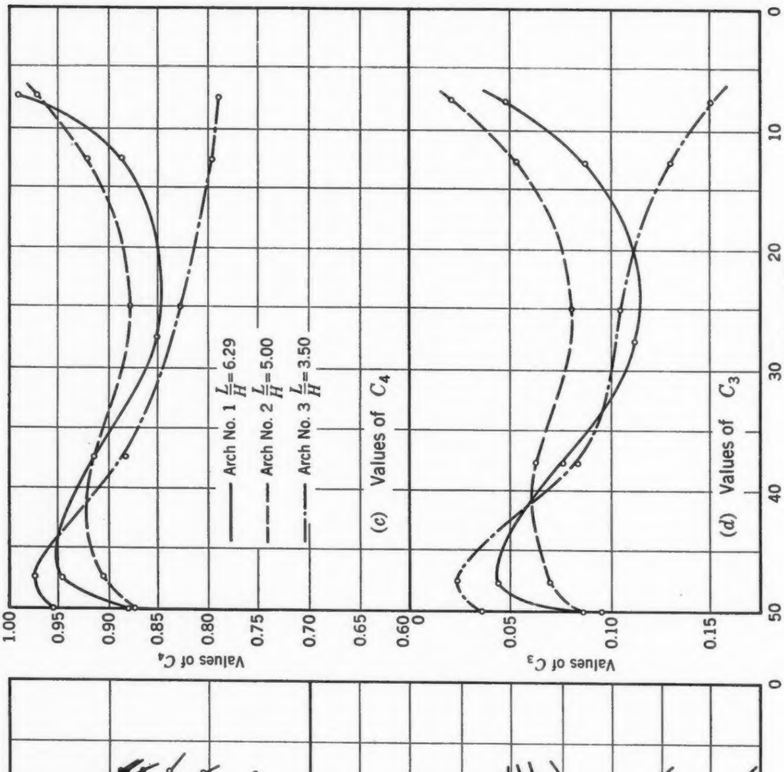

FIG. 18.—CURVES FOR DETERMINING COEFFICIENTS IN EQ. 24

be tabulated for convenience. The value of M_b then may be reconverted to the value for the designer's arch.

It should be noted that the small inconsistencies in the coefficient curves, as plotted from calculations from test results, are not an indication of unreliability in the final equation resulting from their use. The four coefficients are all interdependent, and small changes in one are accompanied by compensating shifts in the others. Eq. 24 and the coefficients have been checked thoroughly by taking values of the coefficients from the various curves for points on the arches between plotted points and by computing the value of M_b for some particular values of $\overline{DL}_b$ and R. This computed value then was plotted with corresponding observed data and found to be entirely consistent.

Accurate results (2%) should be obtained for values of $\overline{DL}_b$ between 2.50 and 7.0 and for all values of R between 1.5 and 15. Use of the equations for other values outside this range will give less accurate results and is not recommended. Interpolating between the coefficient curves for arches with intermediate values of span to rise $\left(\frac{L}{H}\right)$ also is not recommended, as the curves do not vary in a linear manner. The moment M_b for other $\frac{L}{H}$-ratios may be obtained with fair accuracy by computing M_b for all three ratios covered herein, plotting the results against $\frac{L}{H}$, and then selecting the value of M_b for the desired $\frac{L}{H}$ from this curve.

Eq. 24 should offer valuable assistance to a designer in selecting preliminary sections and in checking designs of arches, even if not used for final design.

Dimensionless Plots.—Another useful method of presenting the data is shown in Fig. 19, in which dimensional analysis has been used. It will be noted that the ordinate, $\frac{M}{\overline{DL}\, L^2}$, and abscissa, $\frac{E\, I}{\overline{DL}\, L^3}$, are both dimensionless, since M is in inch-pounds, $\overline{DL}$ in pounds per inch, L in inches, and $E\, I$ in pound-inches². Ratio R is also dimensionless, since it is the ratio of two load intensities in pounds per inch. The points in Fig. 19 were obtained from M on the model, by calculating $\frac{M}{\overline{DL}\, L^2}$ and $\frac{E\, I}{\overline{DL}\, L^3}$ for the corresponding model values of $\overline{DL}$, L, E, I, and by drawing curves of constant R. The ordinates are plotted to logarithmic scale in order to keep the values within reasonable plotting limits and so that the percentage error in plotting would be constant for all values of the ratio R.

The importance of the dimensionless plotting lies in the fact that Fig. 19 applies to any arch that is geometrically similar to the test arch, regardless of size and relation of load to stiffness. For example, suppose a designer has selected certain values of E, I, L, $\overline{DL}$, and $\overline{LL}$ (or R) for his bridge and wishes to find M at a certain point, say near the quarter point, and that his arch has an $\frac{L}{H}$-ratio corresponding to test arch No. 1. Then, he would compute the value of

$\frac{E\,I}{\overline{DL}\,L^3}$, enter Fig. 19 until he reached the appropriate value of R; and, from the corresponding $\frac{M}{\overline{DL}\,L^2}$-value, he would compute the desired M. Care should be exercised to keep all the variables in consistent units (pounds and feet, for example).

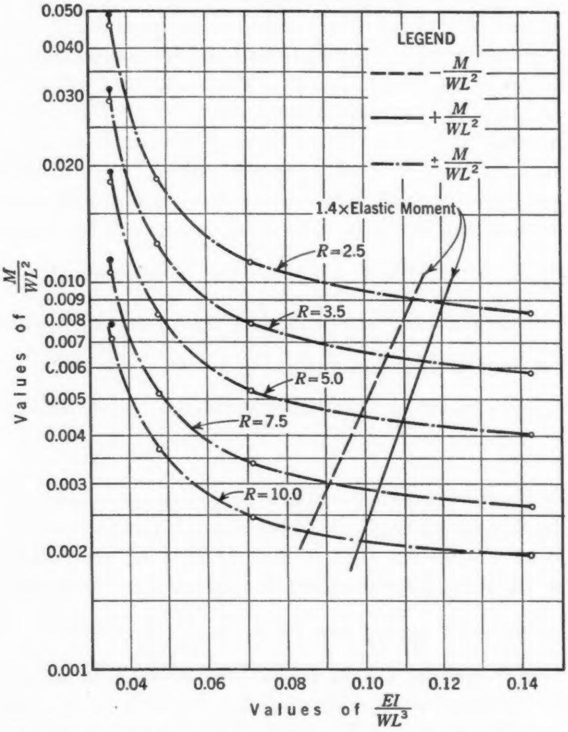

FIG. 19.—DIMENSIONLESS CURVES; ARCH NO. 1, AT POINT 6 C

On the other hand, suppose the designer wishes to investigate the effect on M resulting from a change in $E\,I$. The curves show that an increase in $E\,I$ produces a decreasing effect on the value of M and that $E\,I$ will decrease M effectively only up to a certain point, beyond which thickening the section plays little part in reducing M. In other words, the secondary moments due to deflection become negligible.

In this connection, the elastic-theory moments plot as straight horizontal lines on these dimensionless coordinates, since they are entirely independent of $E\,I$ (as long as it is constant) and $\overline{DL}$. To show how much the plotted curves vary from the elastic-theory values, the points, where the curves are equal to 1.4 times the elastic-theory values, are shown by the two lines in Fig. 19 which intersect the curves of constant R.

Lack of space prevents including the complete set of curves which were calculated for each point on each arch tested.

Curves of Percentage Increase.—The preceding part of the paper has dealt entirely with the absolute values of the moment in the model arches or in arches of geometrically similar shape. This form of presentation is perhaps the most direct, but has certain limitations that can be eliminated by plotting the test results in the form of a percentage increase of observed moment over calculated

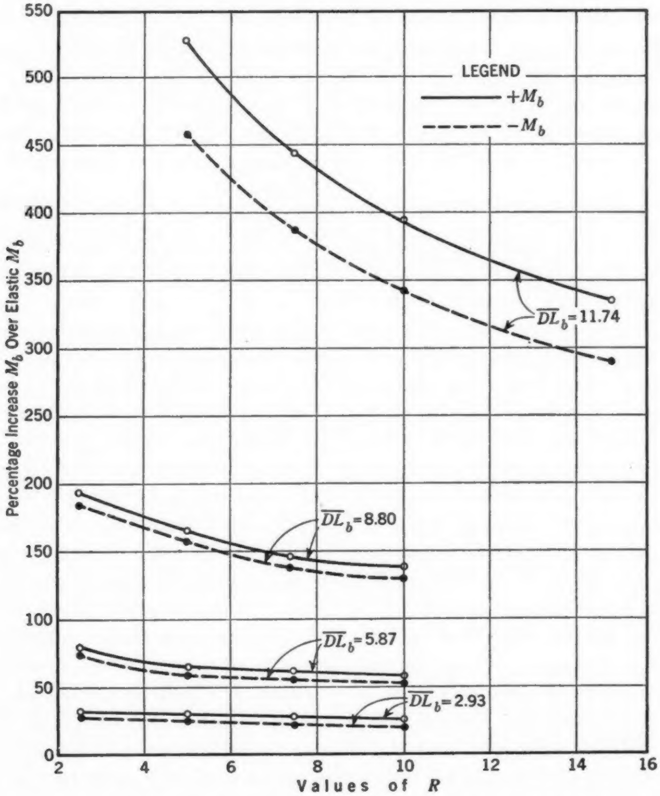

FIG. 20.—PERCENTAGE INCREASE OF ACTUAL M_b OVER ELASTIC M_b, BASE ARCH NO. 1, POINT 6 C

elastic-theory moment. This is done in a set of curves, an example of which is shown in Fig. 20, giving values for approximately the $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{1}{2}$ span points for each of the three base arches. These curves are similar also to Fig. 16 except that percentages are used instead of absolute values for the ordinate. Values for other arches may be obtained readily by use of Eq. 23.

There are three major advantages to such curves:

1. Arches can be classified into three main groups—the first group including those that are stiff enough to permit the use of the elastic theory with no cor-

rections for secondary effects; the second group comprising those arches which can be used satisfactorily, after proper corrections for deviations from the elastic theory have been made; whereas the third group includes those arches which are so flexible as to be entirely unsatisfactory. Conclusions of this nature can be drawn directly from curves of percentage increase, whereas those for actual observed moment must be compared with elastic-theory values to be interpreted.

Most of the tests made in this investigation fall in the second and third categories. For example, assume an arch rib for which $\overline{DL}_b = 5.87$ and $R = 5.00$ and design it in silicon steel for 20,000 lb per sq in. with a direct stress of 10,000 lb per sq in. and a bending stress of 10,000 lb per sq in. If both live load and dead load are increased by 50% so that $\overline{DL}_b = 8.80$ and $R = 5.00$, the direct stress will increase to 15,000 lb per sq in., but, from Fig. 20, the bending moment is now 264.4% of the elastic moment instead of 168.5%. Therefore, the bending stress is now $\frac{2.644}{1.685} \times 1.5 \times 10{,}000 = 23{,}535$ lb per sq in. The total stress would be 38,535 lb per sq in., a stress readily carried by many alloy steels or aluminum alloys. Such an arch would fall in the second category of foregoing arch types.

Had the dead load and live load of the previous example been increased to $\overline{DL}_b = 11.57$ and $R = 5.00$, the direct stress would become 20,000 lb per sq in., but the bending stress would have increased to 74,540 lb per sq in., giving a total of 94,540 lb per sq in. Even if available materials could carry such stresses safely, the deflections coincident with them would rule out such an arch except in very special cases where large deformations could be tolerated.

2. A second use for curves of the type of Fig. 20 is in applying the test results to arches of varying $E\,I$ and to forms other than exactly circular, such as parabolic or three-centered arches. Calculated elastic-theory moments for such ribs are likely to be fairly different from those for circular arches of constant $E\,I$ and of similar general proportions. It is also probable that observed moments of flexible models of such ribs would vary from those on flexible circular arches. However, the percentage increase of actual moment over elastic-theory moment for circular arches of constant $E\,I$ will be almost identical to percentage increases for noncircular arches of variable $E\,I$, provided the variations in stiffness with span and the change in outline are not too pronounced.

Therefore, a designer could use curves of the type of Fig. 20 by: (1) Idealizing his arch into a circular arch of constant $E\,I$ and of very similar proportions; (2) entering the curves to select the proper value of percentage increase; and (3) applying this percentage increase to the calculated elastic moment for the designer's own structure. Thus he could tell readily whether deflection or secondary moments were important in his particular situation.

3. There is a third advantage of the percentage-increase curves. All the test results were for moments caused by a uniform loading of an extent and location such as to produce maximum moment at the observed point. Many highway loading specifications include concentrated loads applied to the arch rib in addition to a uniform load, whereas railroad loadings specify several concentrated loads followed by a uniform load. For such load patterns, the

calculated elastic-theory moment and actual moments are certain to be different from those obtained in these tests. However, it is reasonable to assume that the percentage increase of actual over elastic-theory moment will be approximately the same for any loading pattern. This is particularly true since this increase is largely a function of the relation between $E\,I$ and the dead weight intensity $\overline{DL}$, and not the actual live load intensity (R), as illustrated by the relatively small variation of the curves with R for the majority of the test range.

A small point of interest concerning the curves for percentage increase may be revealed by examination of the curves for the various points across a single arch. Except for the midspan point (10 E) the percentage increase for the corresponding curves at the different span points is found to be substantially the same, particularly for low values of $\overline{DL}_b$.

Application of Data to Niagara Rainbow Arch Bridge

The original two-hinged design of the Rainbow Arch Bridge at Niagara Falls is an excellent practical check on the experimental work, since this arch was a good example of flexibility, and has been studied for deflection moments as described in the preceding paper of this Symposium. By applying the geometrical shape, loadings, and sections of this prototype design to the equations and curves of this paper, the values of moment along the arch can be evaluated and checked with the values as obtained by the designers in their "deflection theory." Furthermore, the calculations which follow more clearly illustrate the use of the experimental data.

The following data were used for the original two-hinged design of the Niagara arch: Span, 950 ft; rise, 150.75 ft; ratio of span to rise $= 6.32 = \frac{L}{H}$; depth of rib, 14 ft (average); I, 119 ft^4; E, 29,400,000 lb per sq in.; dead load $= \overline{DL} = 8.6$ kips per ft at the crown; live load $= \overline{LL} = 1.5$-kips-per-ft intensity; and the ratio $\frac{\overline{DL}}{\overline{LL}} = R = 5.73$.

To illustrate the application of the curves to this arch, the maximum positive moment at the quarter point will be obtained by the use of Eq. 24 and checked with the dimensionless curves and the percentage-increase curves. Since the $\frac{L}{H}$-value for the Niagara arch (6.32) is very close to the experimental value of 6.29 for arch No. 1, the curves and coefficients for this model arch and its base arch will be used.

Eq. 24 is correct in terms of the base arch, so that values of the Niagara arch variables must be converted to the base arch before using the equation. From Eq. 23*a*:

$$\overline{DL}_b = \frac{8.6 \times 29 \times 10^6 \times 100 \times 950^3}{29.4 \times 10^6 \times 119 \times 1{,}000^3}$$

which, in terms of the base arch $= 6.1$ kips per ft. From Fig. 18, for the quarter point of arch No. 1: $C_1 = 0.156$; $C_2 = 1.37$; $C_3 = 0.846$; and $C_4 = 0.115$. Also, $R = 5.73$ (does not change between model and prototype); and, by

Eq. 24:

$$\log M_b = [(0.156 \times 6.1) + 1.37](1 - 0.115 \log 5.73) - 0.846 \log 5.73$$

or

$$M_b = 30{,}200 \text{ ft-kips (30.2 thousand ft-kips)}.$$

Converting this base-arch value of moment back to the prototype, by Eq. 23*b*:

$$M = \frac{30{,}200 \times 29.4 \times 119 \times 1{,}000}{29 \times 100 \times 950} = 38{,}350 \text{ ft-kips.}$$

This value of moment is close to the value arrived at by the designers at working loads, using their "deflection theory," from which the moment was calculated to be 37,130 ft-kips. The elastic-theory value for the same loading condition was 22,600 ft-kips.

The moment can be determined also at approximately the quarter point by using the dimensionless curves, referring to Fig. 19:

$$\frac{E\, I}{\overline{DL}\, L^3} = \frac{29 \times 10^6 \times 144 \times 119}{8.6 \times 1{,}000 \times 950^3} = 0.0685$$

From Fig. 19, interpolating between R-curves for $R = 5.73$: $\frac{M}{\overline{DL}\, L^2} = 0.0047$; from which $M = 0.0047 \times 816 \times 1{,}000 \times 950^3 = 36{,}500$ ft-kips.

The moment at this point (one-half panel toward the center from the quarter point) is not appreciably different from the value at the quarter point.

As a final check the moment will be determined from the percentage-increase curves. Referring to Fig. 20, for $\overline{DL}_b = 6.1$ and $R = 5.73$, the percentage increase of actual moment over elastic-theory moment is 70%. Therefore, the actual moment is $M = 1.70 \times 22{,}600 = 38{,}420$ ft-kips, which is almost identical to the foregoing value calculated from Eq. 21.

Conclusions

1. The experimental results obtained from this investigation, together with the method of presentation, furnish a control and a check on analysis methods for flexible arches which have been developed or may be developed in the future.

2. The data will be of particular assistance in the selection of preliminary or trial sections of two-hinged arch ribs which, in the opinion of the designer, should be sufficiently stiff for practical reasons to make the application of the elastic theory entirely proper.

3. The data supply a basis for the design of relatively flexible, two-hinged arches where such designs are proper—for example, in hanger roofs in which large deflections might not be objectionable if economies can be effected with safety.

4. In flexible arches, the dead load becomes an important consideration when determining moments and any theory which ignores it—such as the elastic theory—is not reliable.

5. Previously published opinions on the effect of reasonable variation of $E\, I$ on the moments in an arch, and of the close agreement between moments in

circular arches and closely similar parabolic arches (in each of which the dead-load resultant follows the center line of the arch) support the contention that the results presented will be of value in the study of arches of noncircular profile and nonuniform section.

6. The relation of load to unit stress in a flexible arch rib is not linear. Accordingly, the safe capacity of such an arch should be estimated by calculating its ultimate (or yield) loads, and then by applying the desired factor of safety to the critical design loads.

7. Conventional methods of calculating temperature stresses are satisfactory for the flexible two-hinged arch rib. (Although not described herein, this matter was checked.)

8. The refined method of analysis developed by the designer of the preliminary two-hinged design for the Rainbow Arch Bridge should yield reliable results.

9. Limited observations were made on the arch models in a fixed-end condition. Although, as might be expected, the secondary moments due to deflections were much smaller than for the two-hinged condition, certain inconsistencies with the elastic theory, of a different sort from those observed on the two-hinged arches, were noted, thus recommending this as a subject for future study.

FABRICATION AND ERECTION CONTROLS

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SYNOPSIS

The hingeless arch ribs were erected by cantilevering symmetrically from each skewback. The first three rib sections were cantilevered from the anchor bolts, a statically determinate condition. Thereafter, the resisting moment at each skewback was augmented by one, two, or three groups of wire-rope strands passing from the forward rib sections to a temporary tower placed behind the skewback, and thence to a concrete anchorage embedded in rock.

A complete description of the erection procedure appears in the fourth paper of this Symposium. It is assumed in this paper that the reader is acquainted with that description. The present paper discusses briefly the shop controls used to insure the correct geometry of the ribs as fabricated and erected.

Throughout the statically indeterminate phases of the erection, it was necessary not only to compute and control the stresses in the permanent steelwork, but to compute the coexisting stresses in the temporary wire-strand tiebacks. This paper describes the procedure using a method of successive approximations for resolving this indeterminacy.

It was also considered desirable to measure, prior to closure, the thrust and moment at the crown of the braced arch-rib sections after they had been made self-supporting by the removal of the wire-strand tiebacks. The thrust and moment thus measured could then be changed, if necessary, by varying the shape of the "keystone" rib section. Thus, the stresses locked in the structure for this loading condition could be controlled, and the effect on the stresses due to accumulated errors in span length, fabricated lengths or angles, skewback angles, etc., could be eliminated. It is believed that this is the first instance of such planned control of crown thrust and moment.

By correcting the shape of the "keystone" rib section properly, agreement of the actual stresses with the calculated stresses for the loads on the arch in this erection stage was assured. However, the fact that no correction was required is an indication of the extreme accuracy of fabrication and pier location.

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Introduction

As described in the first paper of this Symposium, the Rainbow Bridge is a hingeless arch with a span between skewbacks of 950 ft and a rise of 150 ft. As such, the structure is triply indeterminate and the stresses must be calculated from its elastic properties. Initial errors in span survey, in fabricated length or contour, in skewback angle, or in angles at the field joints as riveted could introduce serious errors in the stresses due to the weights in place at the time of closing and swinging, errors that might be an important percentage of the total stresses at some points. Unusual efforts were made, in closing at the crown, to determine and eliminate the accumulated effect of such errors. The fact that they were found to be negligible and did not require correction testifies to the accuracy attained in survey, fabrication, and erection.

The arch center line was established by the designer to follow very closely the dead-load equilibrium polygon, the ordinates thus determined at all panel points being tabulated on the contract plans. The contractor then undertook, for purposes of drafting and fabrication, to pass a compounded series of circular arcs through these points of control.

The spacing between spandrel columns is approximately 40 ft, and the weight of the rib material necessitated a field splice in each rib in advance of each successive spandrel column. A preliminary study of splices determined a distance (usually about 6 ft) from center of column to center of splice which would keep the splice material clear of the column connections. The rib-center radii were then so selected that changes of radius occurred only on radial lines to alternate splice centers. Thus, two successive pieces of rib as fabricated had a constant radius, and changes of radius never occurred within a fabricated piece. The resulting working curve involved five centers on each side, plus one on the vertical axis, the radii varying from 781.6 ft to 970.6 ft. This curve passed through the twenty-five stipulated points with errors not exceeding $\frac{1}{8}$ in.

The specifications required that the arch ribs conform to the stipulated outline under full dead load at 50° F. This required that each rib section be cambered, or increased in length, by the amount that it would shorten under dead load, and that its length be adjusted for the difference in temperature between 50° and 68°, the temperature at which the fabricator's shop tapes are all standardized. This increase in length of each rib section was assumed as a wedge-shaped piece attained by lengthening the top and bottom arc lengths through the angular increment determined by the increment of length on the center line. This meant that, if the complete rib had been shop assembled under no stress, both the chord distance between skewbacks and the angle between radii through the skewbacks would have been greater than the intended values as erected under dead load.

In connection with the problem of camber, the "effective" areas for computing deformations were of considerable importance. It was decided finally to consider the unspliced longitudinal stiffeners as part of the effective area (see Fig. 6), in addition to all spliced material that is effective in resisting stress.

A set of the contractor's erection drawings and calculation sheets, showing the basic outline, arch-rib radii, splice locations, weights of rib sections, effective

areas, and cambering details, has been placed on file at Engineering Societies Library,[16a] together with copies of all sheets recording the computations referred to hereafter. A descriptive index of the filed material is given in Appendix II.

Fabrication

In the fabrication of the individual 40-ft rib pieces, the 12-ft web plates were subdrilled to the intended radii of the top and bottom angle gage lines, and the 8-in. by 8-in by $1\frac{1}{8}$-in. angles and the 54-in. cover plates were subdrilled straight and pulled in, cold, to the required curvature. It was felt that the local bending stresses thus locked in the angles by the riveting would do no harm in a ductile material, whereas "gagging" to curvature would require stressing beyond the yield point and, in all probability, would produce a series of approximate chords rather than a smooth curve. The smooth curve was desirable not only for appearance, but to accommodate the designer's intention that the straight-line bearing edges of the spandrel columns should "rock" slightly on the curved ribs.

To insure that the rib sections would be of the proper depth and curvature, each web with its flange angles was assembled in a jig. The diaphragm plates between the two parts of the box girder were milled on all four edges, and the sections were then boxed in pairs against squaring brackets to obtain and maintain the rectangular cross section. While held against these squaring brackets, the sections were reamed and riveted. They were then removed to the milling machine and milled accurately to length and bevel. While the sections were in position on the milling machine, center lines and working points were established, and the pads for the column bearings were set.

Assembly

After considerable study of the problem of maintaining accuracy of length and angle, the fabricator decided to shop assemble the rib sections on their sides, starting from the skewback and checking the position of each section accurately as it was placed, until seven sections were assembled. After the splices between these sections had been reamed, the first six sections were pulled out ready for shipment, the seventh section was moved back to the starting point, and the remaining five sections were assembled in sequence to the seventh.

According to the terms of the contract, the rib sections were fabricated in the United States, whereas the rib laterals, spandrel columns, and all floor steel were fabricated in Canada. Each fabricator independently prepared metal templates for the common holes—that is, those at the connections of the spandrel columns and the rib laterals to the rib itself. These templates were set, with great precision, on the assembled rib to center lines which had been located while the rib section was in position on the milling machine. Although there was no physical exchange of templates by the two fabricators, the matching of the holes in the field was practically perfect.

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The assembly shop, with four rib assemblies being worked on simultaneously, is shown in Fig. 21. The pieces standing upright at the far end of the shop have been assembled, their splices reamed or drilled, and they are now being painted. Assembling of the piece in the left foreground has also been completed, and the piece removed from the assembly prior to painting. The box sections on the right are the completed grillage and skewback sections, and are loaded on cars ready for shipment. The trapezoidal section in the center foreground is the top section of the cable bent to which the forward ties and backstays attach, standing upside down. Near the center of Fig. 21, a horizontal reamer is shown drilling on a cover-plate splice.

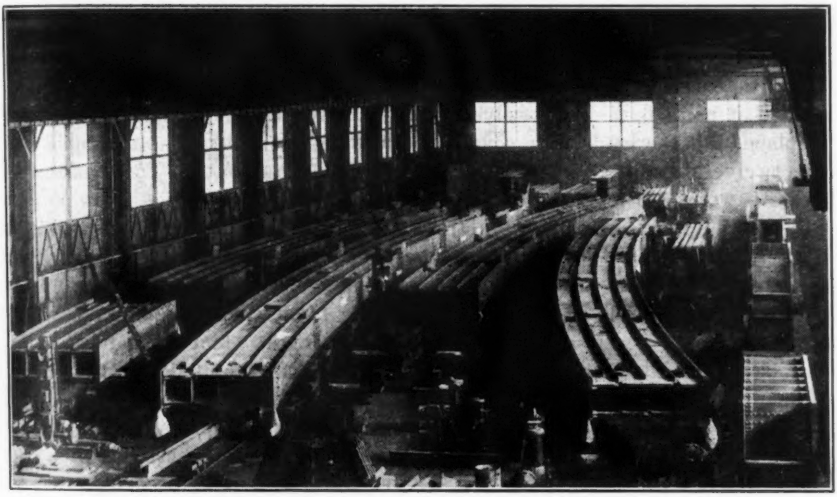


FIG. 21.—VIEW OF FOUR RIB ASSEMBLIES IN THE ASSEMBLY SHOP

Prior to making the aforementioned two types of assembly—seven-piece and six-piece—the offsets to center line were calculated from as many successive base lines as there would be pieces in the final assembly, each base line running from the left-end center of the first piece laid down to the right-end center of the last piece attached. As successive pieces were attached on the assembly skids, the ordinates from the base thus created were measured and compared with the calculations. Thus, curvature was checked in seven different ways through the seven-piece assemblies and in six different ways through the final, six-piece assemblies. Throughout these successive checks, the measured and calculated ordinates were held to differences not exceeding $\frac{1}{8}$ in. While these checks on curvature were being made, the milled ends of successive sections were kept in close contact and the splice holes thus reamed. The care taken in the laying off, milling, and splice-reaming throughout the fabrication resulted in a gratifying accuracy in the finished structure.

The Erection Problem

To assist in an appreciation of the erection engineering calculations required, Fig. 22 is presented to show the successive cantilever positions, and the following summary of successive operations is given.

First, three rib sections (points 13 to 10, Fig. 6) were erected by cantilevering freely from the anchor bolts, a determinate condition. Next, eight wire tie strands per rib were connected at point 10 and put under a measured, predetermined stress. Henceforth, until closure, the conditions were statically indeterminate. Two more rib sections were freely cantilevered beyond point 10, depending upon the splice bolts and pins for stability; eight tie strands were connected at point 8 and put under predetermined stress. The strands at point 10 were then disconnected from the rib but not from the tieback bent. Two more rib sections were cantilevered beyond point 8, and eight tie strands were connected at point 6; these were the strands originally connected at point 10, with extension "pennants" attached. Four additional strands were attached at point 6, making twelve in all. The tie strands at point 8 remained in place, and the loads were shared between the eight strands at point 8 and the twelve strands at point 6 in predetermined amounts. Three more rib sections were cantilevered beyond point 6, and four tie strands from point 6 were successively connected and stressed at point 3; then the eight strands at point 8 were released from the rib, lengthened with pennants, and reattached and stressed at point 3. The loads were shared between the eight strands at point 6 and the twelve strands at point 3 in predetermined amounts. Two more rib sections were cantilevered beyond point 3, which brought the two half arches, cantilevering from the two shores, within the predetermined 11 in. of meeting at the crown.

At the crown, jacking brackets were bolted to the upper and lower cover plates of each half rib, so that two 500-ton jacks could be inserted above, and two symmetrically below, each rib. After these jacks had been operated to pick up a crown thrust and slack it on to shims, all the tie strands were successively released and removed.

Up to this point, it was necessary to predetermine by calculation, for each successive position, the arch-rib stresses, the stresses in tie strands, tieback bent and anchorages, and, from these latter, the necessary initial lengths of the tie strands. At the successive connections of strands to ribs, there were structural bar-and-pin devices to permit shortening and lengthening the strand length in increments of 1 in. to total adjustments of 12 in. and 18 in., respectively, from the mean, and to permit shortening and lengthening by larger increments to maxima of 24 in. and 30 in., respectively, from the mean. With so limited an adjustment possible in long, flexible ties, it was obviously important to determine the unstressed length with accuracy. Ties too short or too long would have seriously changed the stresses in the arch ribs by straining them too far, or not far enough, against their fixation at the skewbacks.

Scope of Computations

To furnish the erection engineer with the data necessary for the proposed erection procedure, rather elaborate computations were required. The work

necessary to obtain this information for each erection stage is outlined herein. The complete tabulation of calculations as prepared by the contractor and checked by the consulting engineers is available at Engineering Societies Library (see Appendix II).

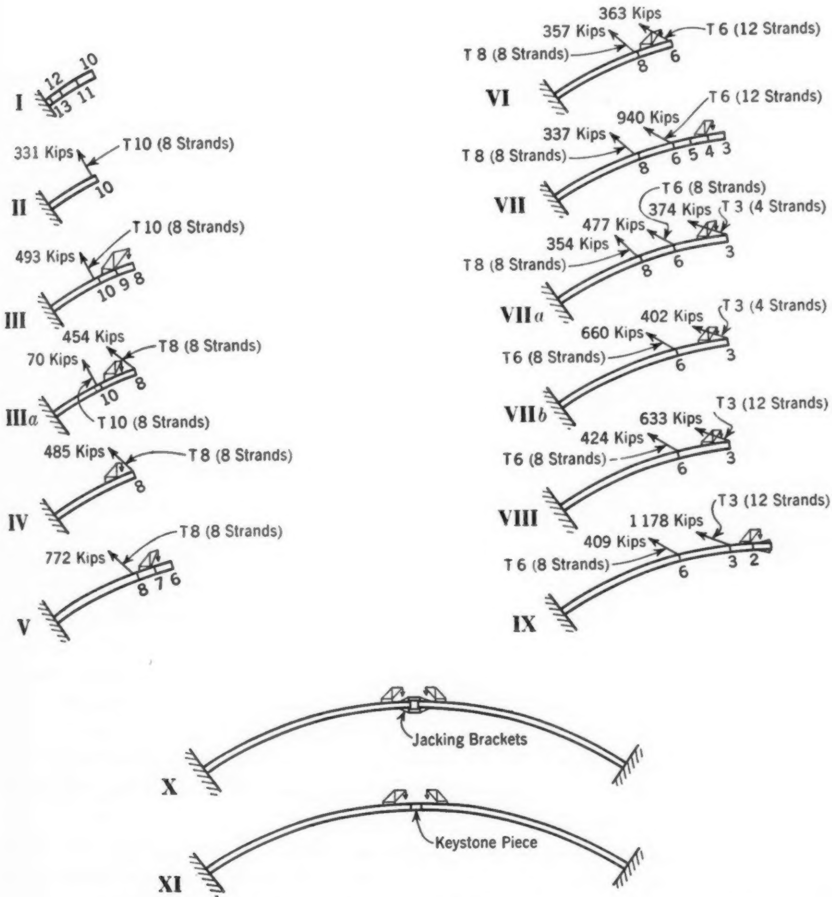


FIG. 22.—ERECTION STAGES AND TOTAL TIE STRESSES FOR ARCH RIB, AT 60° F

The calculation work for each erection stage falls into three divisions, as described under 1, 2, and 3 below. In this description, and in Appendix II, references to sheet or drawing numbers prefixed with ED are to the contractor's erection drawings.

1. (Sheets ED10 to ED15). The part of the rib thus far erected was assumed to be cut loose from any ties and to cantilever from the skewback. Assuming that the steel would hold together and remain elastic under such a

condition, the deflected position was calculated for the points importantly in question—namely, the points of tie attachment and the splice points. Thus, for example, with the rib cantilevered three panels to point 10, the vertical deflection of the free end, in feet, was computed to be:

From bending	0.11406
From thrust	0.00028
From shear	0.00521
Total	0.11955

and the stresses in the rib were found to be just under the allowable. Again, for example, with the rib erected to point 1, but before the jacks and jacking diaphragms at the crown were in place, the hypothetical cantilever deflection was computed to be (in feet):

From bending	49.6117
From thrust	0.0032
From shear	0.1200
Total	49.7349

(and horizontally, 11.9038).

2. (Sheets ED4 to ED8). For a unit vertical force, and for a unit horizontal force, applied at a "tie point" (point of attachment of a wire-strand tie), the displacements, due to the elastic deformations of the rib between that tie point and the abutment, of all rib splice points and all tie attachment points were calculated.

3. In the tie or ties to be attached in the erection stage, forces were first assumed and then changed by the method of successive approximations until their effect, as found from step 2, was: (*a*) To raise the tie attachment points on the rib through approximately the deflected distance found in step 1; (*b*) to stretch the tie system until the end of each tie coincided with its attachment point on the rib; and (*c*) to give safe stresses throughout the rib. An average of four steps in the successive approximations was necessary to get perfect agreement on one erection stage, and this work was doubled since two solutions were made, one using the maximum and the other the minimum expected temperature.

From computations made in this manner, the erection engineer was given not only the information shown in Fig. 22, but also the theoretical elevation of each rib splice point at each erection stage; the stresses in the tie strands; the stress in the first strand to be connected and in the last strand to be disconnected for each forward tie; the effect per inch of change of length of one or all of the adjustment links of each forward tie on tie stress and arch elevation; the influence of change in temperature; and the necessary order for the various steps in each new move.

Displacement Calculations for the Rib

All the theoretical deflections required under step 1, and the displacements due to unit tie force components required under step 2, were computed by the

following adaptation of the "slope-deflection" method, taking separately into account deformations due to bending, thrust, and shear.

The change in angles (Fig. 23(a)) at m in relation to the adjacent members,[16b] due to the bending moment, will be:

$$(\tau_m)' = (M_{m+1} + 2\ M_m)\ \frac{L_m}{I_m} \times \frac{1}{6\ E} \qquad (25a)$$

$$(\tau_{m-1})'' = (2\ M_m + M_{m-1})\ \frac{L_{m-1}}{I_{m-1}} \times \frac{1}{6\ E} \qquad (25b)$$

and the total angle change at m will be:

$$\theta_m = (\tau_m)' + (\tau_{m-1})'' \qquad (26a)$$

These angle changes will accumulate from the supported end, and cause an angle change in the member itself equal to:

$$\phi_m = \sum_{m+n}^{m+1} \theta \qquad (26b)$$

They also will cause deflection at the forward end of the member, with respect to the other end, equal to:

$$u_m = \phi_m\ L_m \qquad (27)$$

and hence total deflections due to bending equal to:

$$\delta_h = \sum u \sin \alpha \qquad (28a)$$

and

$$\delta_v = \sum u \cos \alpha \qquad (28b)$$

Thrust and shear deformation (Figs. 23(b) and 23(c)) will also add up from the support. The total thrust deflections will be:

$$\delta_h = \sum \frac{T\ L}{A\ E} \cos \alpha \qquad (29a)$$

and

$$\delta_v = \sum \frac{T\ L}{A\ E} \sin \alpha \qquad (29b)$$

and the total shear deflections will be:

$$\delta_h = \sum \frac{s\ L}{A_w\ E_s} \sin \alpha \qquad (30a)$$

and

$$\delta_v = \sum \frac{s\ L}{A_w\ E_s} \cos \alpha \qquad (30b)$$

in which A_w = the web area and E_s = the modulus of elasticity in shear.

To apply Eqs. 25 to 30, it was necessary to divide the rib into the thirteen sections defined by the splice points (so as to determine the successive deflected positions of these particular points for field check), and also to subdivide it further at points where there was a change in moment of inertia, and at points of application of concentrated loads (the tie attachment points).

[16b] Correction for *Transactions:* In Fig. 23(a) change ρ to u in three places; and in Fig. 23(c) change S to s in four places.

The calculation record sheets (ED10 to ED15) for the rib deflections required by step 1, under the heading "Scope of Computations," may be typified by Table 5, which is a résumé of sheet ED10, applying to erection stages I and II, Fig. 22.

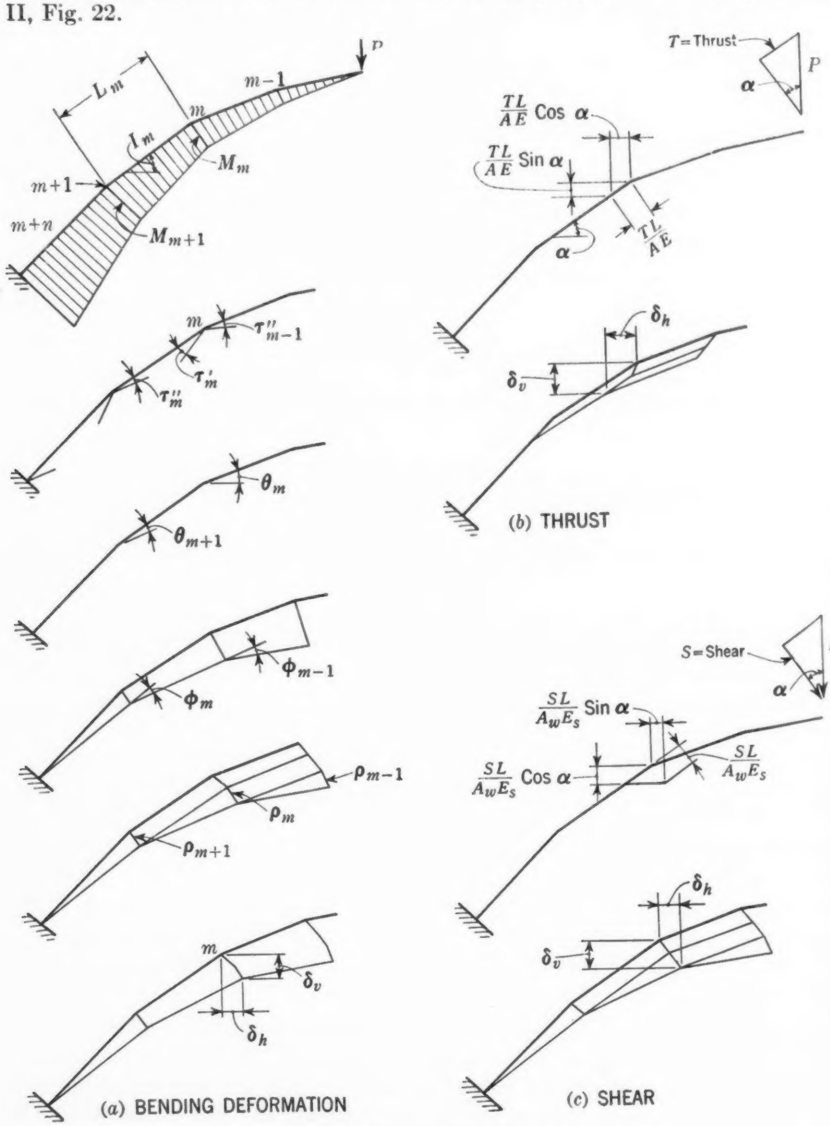


FIG. 23.—DEFORMATION DIAGRAMS

Sheets ED4 to ED8, covering computations for rib displacements due to unit vertical and unit horizontal force at a tie point, were similar. Because the calculations were made for unit loads instead of for steel-and-traveler dead

loads, certain initial column headings differed. For example, in these rib-displacement computations, Figs. 24(*a*), 24(*b*), and 24(*c*), respectively, replaced Cols. 5 to 9, 20, and 28, Table 5. (Note that, in Fig. 24(*a*), 475.00 is the horizontal distance from the midspan to the skewback, SB; and 374.84 is the horizontal distance from the midspan to panel point 10.) Also, the deflections from unit load being very small compared to those for actual loading, the powers of 10 in all columns of sheets ED4 to E8 were changed, and the resulting deflections were obtained in integral numbers of millionths, instead of thousandths, of a foot.

Point	(5)	(6)	(17)	(25)
		1 (X − 374.84)	1 × sin α	1 × cos α
	X	M	P	V
	Ft	Kip-ft	Kips	Kips
SB	475.00	100.16		

(*a*) (*b*) (*c*)

FIG. 24.—EXAMPLE OF ALTERNATE COLUMN HEADINGS FROM SHEETS ED4

From the theoretical deflected position of the rib in any erection stage, from the restoring deflections available from unit vertical and unit horizontal tie pulls, and from the approximate angle of the tie after restoring the arch to almost its proper elevation, the tie stresses could now be approximated. It was found that these agreed sufficiently well with much cruder computations made before compiling the bidding estimate. Since the $1\frac{9}{16}$-in. strands had a guaranteed strength of 310,000 lb each, it was hoped to keep working loads within 100,000 lb, but it was considered proper not to change the assumed number of strands if the final calculations should indicate 120,000 lb. The final maximum (see Fig. 22, stage VII-B) was just over 100,000 lb. Actually, the cable stress for the condition in stage V was also slightly greater than 100,000 lb but only for the minimum temperature condition. The modulus of these strands was assumed as 24,000,000 lb per sq in., that value having given satisfactory results when the strands were previously used as footwalk support ropes at the Tacoma Narrows Bridge in Washington.

DISPLACEMENT CALCULATIONS FOR THE TIES

All calculations for balancing the cable stress and tie positions with the corresponding data for the rib itself were made by an exact method which involved a modified form of the "deflection theory" for suspension bridges. This was necessitated by the extreme flexibility of the interlocked system. The methods and formulas used would be grasped quickly by any designer conversant with cable calculation work, but are of such a highly complex and specialized nature that in their stead the method of approach will be given in general terms.

A diagrammatic sketch of the tieback arch with the loads in position is shown in Fig. 25. These loads comprised: The weight of the arch rib, the uni-

form weight of the backstay and of the forward-tie strands, the weight of the links at the bent and at the rib, the eccentric weight of the bent with its back-leg, and the concentrated load introduced on the forward-tie strands by the pennant sockets when there were any.

In the first instance, three "basic" cases were computed to determine the length to which the several strands should be socketed. These "basic" cases were those of maximum stress in the strands and, in order of attack, were:

(1) Erection stage IX, ties at T6 and T3, arch erected beyond T3 to crown;
(2) Erection stage V, ties at T8, arch erected beyond T8 to T6; and
(3) Erection stage III, ties at T10, arch erected beyond T10 to T8.

TABLE 5.—Displacements Under Dead Load, for Canti

Point	(a) Rib Properties				(b) Bendin[g]						Deforma[tion]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
								(5) × (7)	Σ(8)		(1)/(2)
	L	$I\times10^{-3}$	sin α	cos α	X'	F	ΣF	Q	M	M' and M''	$\frac{L}{I}\times 10^3$
	Ft	In²-ft²			Ft	Kips		Kip-ft			1/(in²-ft)
SB						54.0			24,078		
	9.25	51.26	0.5497	0.8354	7.73		489.1	3,781		68,453	0.180
SP13						114.4			20,297	64,672	
	17.30	22.14	0.5331	0.8461	14.64		374.7	5,486		55,405	0.781
·									14,811	49,919	
	11.74	20.62	0.5331	0.8461	9.93		374.7	3,721		40,712	0.569
SP12						155.5			11,090	36,991	
	15.04	21.11	0.5011	0.8654	13.02		219.2	2,854		30,416	0.713
·									8,236	27,562	
	10.00	19.26	0.5011	0.8654	8.65		219.2	1,896		22,812	0.519
·									6,340	20,916	
	18.53	17.81	0.5011	0.8654	16.04		219.2	3,516		15,504	1.040
SP11						146.4			2,824	11,988	
	23.05	15.78	0.4606	0.8876	20.46		72.8	1,489		6,983	1.461
·									1,335	5,494	
	10.93	13.72	0.4606	0.8876	9.70		72.8	706		3,299	0.797
·									629	2,593	
	8.01	∞	1.0000	0	0		0	0		1,258	0
TP10									0	629	
	9.73	13.72	0.4606	0.8876	8.64		72.8	629		1,258	0.709
SP10						72.8			0	629	
										0	

These "basic" cases were computed at +60° F for ordering strand lengths. They and all other cases were computed at +60° F and −10° F for determination of arch position, and for a check against any overstress of ribs or strands within this temperature range.

In the first "basic" case investigated (erection stage IX at 60° F), it was assumed that the bent was vertical. This condition was nearly maximum for stress in the backstays (−10° F would have been maximum), and was used for ordering the backstay strands and the full-length forward ties at T6. It also determined the total length of the T10-plus-pennant ties which were used at T6, and of the T6-plus-pennant and T8-plus-pennant ties which were used at T3.

Cantilever Condition; Stages I and II

Deformation									Point
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	
(1)/(2)	(10) × (11) × 10^{-3}		Σ(13)	(14) × (1) × 10^{-3}	(15) × (3)	(15) × (4)	Σ(16)	Σ(17)	
$\frac{L}{I} \times 10^3$	$6\,E\,\tau$	$6\,E\,\theta$	$6\,E\,\phi$	$6\,E\,u \times 10^{-3}$	$6\,E\,u \times 10^{-3} \times \sin\alpha$	$6\,E\,u \times 10^{-3} \times \cos\alpha$	$6\,E\,\delta_h \times 10^{-3}$	$6\,E\,\delta_v \times 10^{-3}$	
1/(in.²-ft)	Kips per sq in.			Kip-ft per sq in.					
		12.3					0	0	SB
	12.3								
0.180									
	11.6								
		54.9			(etc.)				SP13
	43.3								
0.781					....				
	39.0								
		62.2			....				·
	23.2								
0.569					....				
	21.0								
		42.7			....				SP12
	21.7								
0.713					....				
	19.7								
		31.5			....				·
	11.8								
0.519					....				
	10.9								
		27.0			....				·
	16.1								
1.040					....				
	12.5								
		22.7			....				SP11
	10.2								
1.461					....				
	8.0								
		10.6			....				·
	2.6								
0.797					....				
	2.1								
		2.1			....				·
	0								
0					....				
	0								
		3.0			....				TP10
	0.9								
0.709									
	0.4								
		0.4					11.17	20.12	SP10
	0								

TABLE 5.

Point	(c) THRUST DEFORMATION								(d) SHEAR DEFORMATION			
	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)
	(7) ×(3) $\times 10^{-3}$	(20) ×(1)		(21)/(22) $\times 10^{-3}$	(23) ×(3)	(23) ×(4)	Σ(24)	Σ(25)	(7) ×(4) $\times 10^{-3}$	(28) ×(1)	Web area	(29)/…
	$P \times 10^{-3}$	$P L \times 10^{-3}$	A	$\frac{P L}{A}$	$P L \times \sin\alpha \div A$	$P L \times \cos\alpha \div A$	$E\,\delta_v$	$E\,\delta_h$	$V \times 10^{-3}$	$V L \times 10^{-3}$	A_w	$\frac{V L}{A_w}$
	Kips	Kip-ft	In.²	Kip-ft per sq in.					Kips	Kip-ft	In.²	
SB							0	0				
(etc.)												
....												
....												
....												
SP10							8.22	13.75				

The second "basic" case determined the order length of the strands used at T8, and the third determined that of the strands used at T10. In these stages the backstay stress was much less than in stage IX, and the bent leaned shoreward.

When the backstay strands were first erected, before any forward ties were in place, the bent had to be supported by a back-leg. To keep the initial stress in the backstay strands within the capacity of the pulling tackle which was used in their initial adjustment, the top of the bent was set back 3 ft. When the load from the T10 cables came on, the bent was pulled forward, of course, releasing the back-leg, and, from that time until final dismantling, it was pin-ended at the base.

In the calculations for the three "basic" positions, the problem was to determine a length of tie such that, when the horizontal and vertical components of the force at the point of attachment F (Fig. 26(*a*)) were applied first to the rib and then to the tie, the resulting location of F as determined from the tie system would coincide with that as determined from the rib.

The first step was to determine a position of the rib for each "basic" case, and to compute *H*-forces and *V*-forces (interrelated by an assumed slope of the link at its connection to the rib) which would raise the free-cantilevered rib to this position. The determination of this position, which was always at or near

E 5.—(*Continued*)

(d) Shear Deformation				(e) Summary of Displacements, in Feet (Thousandths)								Point
(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)	
(31) ×(3)	(31) ×(4)	Σ(32)	Σ(33)	(18) $\times 10^3$ $\div 6E$	(27) $\times 10^3$ $\div E$	(34) ×2.6 $\times 10^3$ $\div E$	Σ	(19) $\times 10^3$ $\div 6E$	(26) $\times 10^3$ $\div E$	(35) ×2.6 $\times 10^3$ $\div E$	Σ	
$VL \times \sin\alpha \div A_w$	$VL \times \cos\alpha \div A_w$	$\frac{E}{2.6}\delta_h$	$\frac{E}{2.6}\delta_v$	Horizontal				Vertical				
Kip-ft per sq in.				Bending →	Thrust ←	Shear →	Total →	Bending ↓	Thrust ↓	Shear ↓	Total ↓	
				0	0	0	0	0	0	0	0	SB
												(etc.)
												....
												....
												....
		34.58	58.89	63.32[a]	0.47[a]	3.06[a]	65.91[a]	114.06[a]	0.28[a]	5.21[a]	119.55[a]	SP10

[a] These quantities are the final results applicable to SP10, the free end in erection stage I, and will be recognized as including those stated in the text under "Scope of Computations."

"normal" or "geometric" position (namely, that which the point would occupy after completion of the entire structure), involved a consideration of the action of the rib under the then existing loading, and under subsequent loadings, at various temperatures.

In the first "basic" case, the elevations of the tie connection points T3 and T6 were determined by finding their location for erection stage IX (Fig. 22) such that the crown opening would be near "normal" at 60° F and the rib moments would be within safe limits. These tie-point elevations were determined for assumed loads from the cables at assumed slopes, and were essentially unchanged under the exact loads and slopes finally found; it was not essential that the final position in erection stage IX be exactly as stated, since it was important only that the ribs at closure be far enough apart to permit erection of the final members and insertion of the jacks. A final check was made of the crown elevation and opening, of course, after the tie lengths had been accurately determined, and, if it had been found that the elevations of the tie points originally used had led to an impossible closing condition, the tie lengths would have been recomputed for new assumptions of elevations of the tie points.

In a similar manner, the "basic" elevation of T8 was, in the second "basic" case, determined so that, starting from this elevation in erection stage V, the succeeding stages of erection could be completed without requiring that the

ties at T8 be readjusted after their initial adjustment had been made. The "basic" elevation of T10 was determined similarly for erection stage III.

It must be noted, however, that the choice of the "basic" elevations thus selected also affected the stresses in the ribs, so that, particularly in the initial

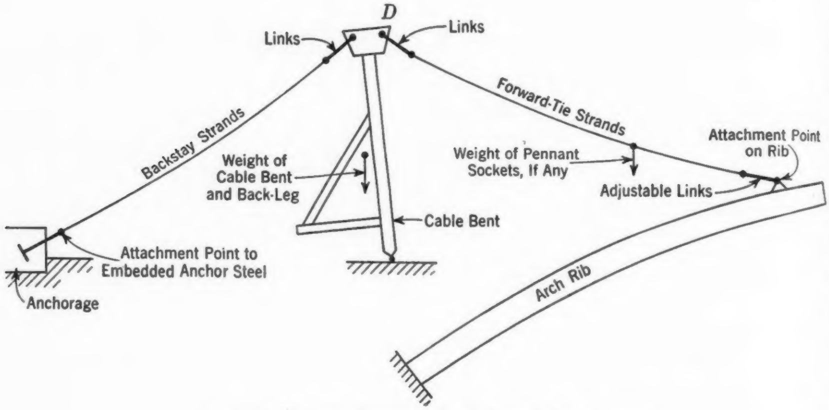


FIG. 25.—LOADS CONSIDERED IN CABLE CALCULATIONS

erection stages when the rib was very stiff and resistant to change in elevation, it was necessary to investigate and modify the "basic" tie-point elevations to keep the rib stresses within allowable values.

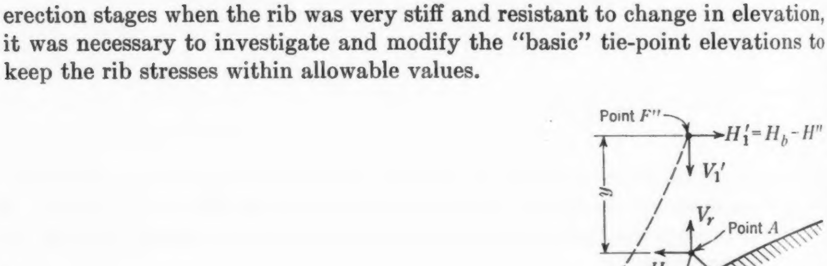


FIG. 26

Assuming a trial value of H, a length of tie was computed which would bring the tie and rib into coincidence at point F, and, for this length, the value of V for the tie was computed (that is, the slope of the link at the rib was computed, thus automatically determining V). By successive approximations, modifying

H and V on the rib each time but holding the elevation of point F essentially constant, tie lengths were finally determined for forces H_b and V_b which balanced the system. These lengths were the "basic" lengths, and the final position of point F so determined was the "basic" position of the point of attachment of the link to the rib.

By using these "basic" results as a starting point, and with the strand lengths now established, the amount of successive approximation computation for the remaining or "intermediate" cases was considerably reduced. These "intermediate" cases comprised erection stages II, IV, etc., and all stages at which strand ties were added, adjusted, or released. With one minor exception, the strand stresses in each "intermediate" case were less than those in the "basic" case to which it was immediately related (as stage II to stage III, etc.).

The method used for calculating the position and H-forces and V-forces for any intermediate condition is shown graphically in Fig. 26(b). With the location of point F, and the "basic" forces H_b and V_b known, values of H_r and V_r were assumed which would raise the rib to point A, approximately its normal position. This H_r-force was then applied to the link and cable system (the "basic" position of the cable was actually used so that only a differential force $H' = H_b - H_r$ was applied to this system), and a horizontal deflection Δ was determined so that point F moved horizontally to point F′; V_1 was then the vertical force required to balance the cable. Since points F′ and A did not coincide, point F′ was moved to point F″ by considering the forward-tie assembly rotated about the point of attachment of the link to the bent, and a new and slightly different $(H_1)'$ and $(V_1)'$ were determined. The vertical distance y was then a measure of the error in the assumption originally made, and new assumptions were set up and the process repeated as many times as was necessary to make points F″ and A coincide. After the cable system and rib were mutually in equilibrium, the moments and shears throughout the erected portion of the rib were investigated to assure that there were no overstresses.

The problem of connecting and disconnecting sets of forward ties and of adjusting them to stress or elevation made it desirable to investigate the effect of changing the length of all strands by 1 in., and of changing the length of one strand by 1 in. When more than one set of ties was connected to the rib at one time, as was the case beginning with erection stage V, calculations for the unit change in length of one set of strands included not only the effect on the stress in these strands and the elevation of the rib at their point of attachment, but also the same effects on all other effective sets of strands. These results were tabulated for field use, so that the erection forces had complete information regarding elevations of the rib at all splice and tie points, stresses in the strands, and effect of changing strand length on both elevation of the rib and strand stress, for all erection stages shown in Fig. 22. They also were given a sequence for releasing, partly, the load on the strands by jacking the ribs apart at the crown, casting off some strands, modifying the jack load, and finally casting off the remaining strands. Thus, it was possible to cast off the strands progressively, without any intermediate partly released stages, and without danger of excessive loads on top or bottom shims at the crown, or of excessive stresses on the last strands left in place.

Field Control of Tie and Rib Stresses

To measure the stresses in the tie strands, the contractor provided himself with four patented dynamometers, the principle of which is as follows: Holding the end supports (which are about 4 ft apart) upon the stressed rope, a spring is released which imposes a constant force transversely upon the rope, midway between supports. Since the deflection of the rope produced by this transverse force bears an inverse relationship to the longitudinal stress in the strand, the stress in the rope may be read on a properly calibrated dial. At the top of the tieback bent, platforms were installed on which the contractor's engineers could operate these dynamometers, free from the confusion and danger on the bridge.

If the external loads applied to the ribs by the forward ties were known for any erection stage, the stresses in the rib were no longer indeterminate. The aforementioned dynamometer provided a means for determining the actual loads applied to the rib by the forward ties, and thus made it possible, in the initial erection stages, to adjust the strands to a precalculated load so that the rib would not be overstressed. In the initial erection stages it was not practicable to determine the load applied by the ties by measurements of the rib deflection, since the rib was so stiff that large changes in tie loads produced deflections of the rib too small to measure. However, in later erection stages, the opposite effect occurred; the rib was then so flexible that even very small changes in tie loads produced large deflections. Therefore, in these later stages, elevations on the rib were used as the criterion in determining the forward-tie strand adjusted length, rather than the loads as measured by the dynamometer, and these loads were used only as a check.

This condition was actually a fortunate occurrence, as it happened that this strand-load check proved to be somewhat illusory. In the later stages, with the ribs at correct elevations, the sum of the strand forces was read 10% or more above the computed values. It was felt that the computations could not be in error to such a degree. The possibility to be feared was not that the strands would yield at these stresses, but that the dead loads had been underestimated and therefore the stresses in the finished bridge would be greatly in error. The contractor rejected this hypothesis and confidently maintained that the dynamometer readings were in error. Later, when the crown thrust was measured on the jacks and found to be exactly as computed, there was general satisfaction. In the initial stages, where dynamometer readings were used as the criterion, it is believed that they were in error by no more, and probably less, than 5% since the angle of inclination of the ties with the vertical was small, thus reducing the correction factor for slope.

The apparent error in dynamometer readings may have been in the calibration or in the manipulation, or in both. The dynamometers are understood to have been positively calibrated in a vertical rope-testing machine, and in a horizontal strand-pre-stressing machine. Correction charts were provided for inclined strand positions, such as prevailed on this operation, and it is natural to question their accuracy. Also, it was found that methods of manipulation, and the particular points on the strand to which the bearings were applied, changed the readings considerably. It follows that the technique used in

taking the readings, which were recorded, may not have been the technique that should have been selected.

The adjustment of the several strands at one tie point was tedious, usually consuming several working days. The accuracy of the strand-length computations was such that there always was more than enough in the pinned links to put the rib at proper elevation. Sometimes one or more strands in a group would be connected at a 1-in. interval longer or shorter than most of the group, so that the measured stresses could be averaged better. To save time, it was considered permissible to let one or more strands take 15% more load than the average of the group, because that did not exceed the 120,000 lb considered proper. Levels taken along the ribs after each adjustment showed the arch-rib contours to be following the prescribed curve within fractions of an inch at all points.

The Elastic Centroid—Its Application to the Study of the Action of the Closed Rib

To study the stresses in the arch ribs, with the half arches erected to the crown but not connected, and the method of controlling these stresses through jacking thrusts, it was convenient to use the previous type of analysis, with modifications. The previous concept of a rib deflected from the skewback as a cantilever was retained, and the imaginary cantilever deflections from bending, thrust, and shear were found as before. However, instead of introducing forces at the forward-tie points to raise the deflected half rib to its proper position, it was necessary to raise each half rib by applying to it, at the crown, a thrust and a moment coming from the opposite half rib. Since loads were symmetrical about the vertical axis through the crown, there would be no transfer of shear. Each half rib was assumed to carry, in addition to its own weight, the erection travelers and equipment in their final position and the jacks, including their supports and appurtenances, but no tiebacks.

The physical forces available and under control at the crown would be a thrust from two 500-ton jacks above each rib and a thrust from two equal jacks below each rib. These jacks would be equidistant from the rib center line, and the manipulation of one set would be independent of the manipulation of the other. Therefore, both thrust and moment were alterable at will, provided that the effects of opening and closing the upper and lower jack openings, in any combination, could be known in advance in terms of change of thrust and change of moment.

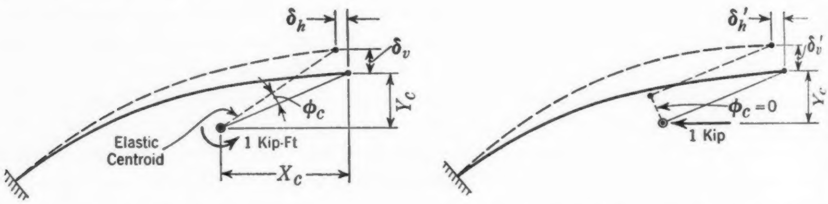
Also, it was necessary to know the effect of jack thrust upon the crown elevation of the arch. Increasing the thrust would spread and raise the half ribs, and such an adjustment might have become desirable if the two arches were somewhat out of level across the bridge. It was not the intention to adjust the crown opening to change the crown elevation of the bridge as a whole, as the ribs were relatively flexible, and any substantial vertical change would require too much change of the crown opening to be practicable.

Another desideratum in the crown adjustment was that the two half ribs should be left in such a relationship, as to distance and angle at the crown, that

the prefabricated floor panel 1–1′, several feet above the ribs, should fit without alteration of its over-all length.

To study these effects of jacking thrusts at the crown, the concept of the "elastic centroid" was introduced.[17] The method here used, based on the physical concept of the elastic centroid defined in this paper, was presented by H. Müller-Breslau in 1925. The subject of the elastic centroid, also called the elastic center or neutral point, is approached from a somewhat different point of view in texts written in English,[18] but the basic concept is the same.

The elastic centroid of the half arch is a point, located mathematically, where the effect of thrust and moment upon the half arch may be segregated; it is a point in space, but is assumed to be connected to the crown point of the half arch by an infinitely rigid member. A unit moment applied at the centroid produces no translation of the centroid, but does rotate the imaginary connecting arm. A unit thrust (a horizontal force) applied at the centroid produces translation of the elastic centroid (and crown of the arch), but no rotation of the imaginary connecting arm or of the arch itself at the crown. This is shown graphically in Fig. 27.



(a) UNIT MOMENT AT CENTROID (b) UNIT HORIZONTAL FORCE AT CENTROID

FIG. 27

The problem of determining the thrust, moment, and shear existing at any point in any hingeless arch, of course, is one involving the analysis of an indeterminate structure. The general rule for such analysis is to cut the structure at the point in question, thus dividing it into two determinate structures. By evaluating the forces required to bring the cut ends into coincidence, the internal forces in the uncut structure can be determined.

Closure of the Rainbow arch was made at the crown, and all calculations were based on cutting the structure at that point. Since the structure and erection loads were symmetrical, consideration of shear was eliminated. It was only necessary to determine for one arm the thrust and moment, at the crown, that would return the freely deflected arm to its closed position—erection loads, camber, and temperature being considered.

It is obvious that both the crown moment and crown thrust produce rotation and thrust at the crown. The analysis of these interrelated functions is complex and is greatly simplified by the introduction of the elastic centroid concept in which translation and rotation are segregated. The procedure used in de-

[17] "Die graphische Statik der Baukonstruktionen," by H. Müller-Breslau, Vol. 2, Pt. 2, 1925, p. 377 *ff.*

[18] "Theory of Modern Steel Structures," by L. E. Grinter, The Macmillan Co., New York, N. Y., Vol. II, 1937, p. 206 *ff.*; also, "Movable and Long-Span Steel Bridges," edited by G. A. Hool and W. S. Kinne, McGraw-Hill Book Co., Inc., New York, N. Y., 1923, p. 440 *ff.*

termining the thrust and moment at the centroid, and in translating these functions to the crown, follows:

To establish the location of the centroid, a moment of 1 kip-ft was applied to the arch rib at the crown, and ϕ_c, δ_h, and δ_v were computed for the crown from Eqs. 25 to 28, where (taking into consideration the basic properties of the rib), as defined by Fig. 27(*a*):

$$\phi_c = \frac{222.750}{6\,E\,10^3} = 0.001263 \times 10^{-3} \ldots\ldots\ldots\ldots\ldots (31a)$$

$$\delta_h = 0.0566 \times 10^{-3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (31b)$$

and

$$\delta_v = 0.2854 \times 10^{-3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (31c)$$

For the purpose of simplifying computations, all calculations involving E or E_s (such as Eq. 31*a*) were made in kip units (E = 29,400 kips per sq in. and $E_s = \frac{E}{2.6} = \frac{29{,}400}{2.6}$ kips per sq in.). After solving Eqs. 31, the centroid ordinates were then (see Fig. 27(*a*)): $X_c = \frac{\delta_v}{\phi_c} = \frac{0.2854}{0.001263} = 226.0$ ft; and $Y_c = \frac{\delta_h}{\phi_c} = \frac{0.0566}{0.001263} = 44.8$ ft.

In skeleton form, Table 6 gives the computations (from sheet ED9) leading to Eqs. 31. A horizontal load of 1 kip at the centroid, using Eqs. 25 to 30, produced bending, thrust, and shear deformations. As defined by Fig. 27(*b*):

$$(\delta_h)' = (2.108 + 0.025 + 0.019)\,0.10^{-3} = 2.152 \times 10^{-3} \text{ ft}$$
$$(\delta_v)' = (6.557 - 0.007 + 0.047)\,0.10^{-3} = 6.597 \times 10^{-3} \text{ ft}$$

Table 7 gives in skeleton form the computations (from sheet ED9) leading to the foregoing numerical results. In the complete table (sheet ED9) from which Table 7 and other similar tables are condensed, the "point" column contains an entry for each splice point, SP13 to SP1, inclusive, and for each intermediate point at which there is a change in the area, and the moment of inertia, of the rib.

If a vertical force P is applied to the half rib at some point A, and if δ_v is the vertical deflection at point A due to a unit moment at the centroid (or by Maxwell's law, the rotation at the centroid due to unit vertical load at point A), then the angular rotation at the centroid due to force P is $P\,\delta_v$. The restoring moment at the centroid to overcome this angular rotation is $\frac{P\,\delta_v}{\phi_c}$ in which, by Fig. 27(*a*), ϕ_c is the rotation at the centroid due to unit moment. These values of δ_v for all critical points on the rib were computed for unit moment at the centroid. The value of ϕ_c is given in Eq. 31*a*.

Similarly, if the same force P is applied at the same point A, and if $(\delta_v)'$ is the vertical deflection at point A due to a unit horizontal force at the centroid (or, by Maxwell's law, the horizontal deflection at the centroid due to unit vertical load at point A), then the horizontal deflection at the centroid due to force P is $P\,(\delta_v)'$. The restoring horizontal force at the centroid to overcome this deflection is $\frac{P\,(\delta_v)'}{(\delta_h)'}$ in which $(\delta_h)'$, by Fig. 10, is the horizontal deflection

at the centroid due to a unit horizontal load at the same point. These values of $(\delta_v)'$ were computed for all critical points on the rib for a unit horizontal load at the centroid. The value of $(\delta_h)'$ is 2.152×10^{-3}, as stated.

Thus, for any series of vertical loads acting on the half rib, the foregoing computations offer a rapid and accurate method for determining the moment

TABLE 6.—DISPLACEMENTS DUE TO UNIT MOMENT OF 1 KIP-FT (SEE FIG. 27(*a*))

Point	(*a*) RIB PROPERTIES					(*b*) BENDING			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
								(7) × (3)	
	L	$I \times 10^{-3}$	$\frac{L}{I} \times 10^3$	sin α	cos α	M	M'' and M'	$6\,E\,\tau \times 10^3$	$6\,E\,\theta \times 10^3$
	Ft	In²-ft²	1/(in²-ft)			Kip-ft		Kips per	
								....	
SB						1			0.540
							3	0.540	
	9.25	51.26	0.180	0.5497	0.8354				
							3	0.540	
SP13						1			2.883
							3	2.343	
	17.30	22.14	0.781	0.5331	0.8461				
							3	2.343	
·						1			4.050
							3	1.707	
	11.74	20.62	0.569	0.5331	0.8461				
							3	1.707	
SP12						1			3.843
							3	2.136	
	15.04	21.11	0.712	0.5011	0.8654				
							3	2.136	
(etc.)									
....									
....									
....									
							3	11.571	
SP1						1		....	11.571
								$6\,E\,\phi_c \times 10^3 =$	

and thrust, at the centroid, required to bring the crown to a position such that it can be joined to the adjacent half rib. The requirement is, of course, that the crown point have zero horizontal movement and zero rotation. This moment and thrust was:

$$M = \frac{\Sigma P\,\delta_v}{\phi_c} \quad \ldots\ldots\ldots\ldots (32a)$$

and

$$H = \frac{\Sigma P\,(\delta_v)'}{(\delta_h)'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32b)$$

The thrust at the elastic centroid is also affected by the shop cambering and by temperature; the moment at the elastic centroid is not affected by changes in

APPLIED AT THE ELASTIC CENTROID (HALF ARCH), IN THOUSANDTHS OF A FOOT

Deformation								
(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	Point
Σ(9)	(10) × (1)	(11) × (4)	(11) × (5)	Σ(12)	Σ(13)	(14) ÷ 6 E	(15) ÷ 6 E	
$6E\phi \times 10^3$	$6Eu \times 10^3$	$6Eu \times 10^3 \times \sin\alpha$	$6Eu \times 10^3 \times \cos\alpha$	$6E\delta_h \times 10^3$ ←	$6E\delta_v \times 10^3$ ↑	$\delta_h \times 10^3$ ←	$\delta_v \times 10^3$ ↑	
sq in.	Kip-ft per sq in.					Ft		
				0	0	0	0	SB
0.540	5	3	4					
				3	4	0.000017	0.000023	SP13
3.423	59	31	50					
								.
7.473	88	47	74					
				81	128	0.00046	0.00073	SP12
11.316	170	85	147					
								(etc.)
								....
								....
								....
				9,983	50,350	0.05659	0.28543	SP1
222.750								

length along the rib. If δL is the camber change in length of one section of the rib, α the angle of this section with the horizontal, and $(\delta_h)' = 2.152 \times 10^{-3}$, then the thrust due to camber is

$$H_c = \frac{\Sigma \delta L \cos\alpha}{(\delta_h)'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

Table 8(*a*) gives, in skeleton form, the computations (from sheet ED9) for the calculation of thrust due to increase of length in cambering, and the vertical displacement (upward) resulting from the elimination of spread by the pressing of one crown on the other.

If t is the change in temperature in degrees Fahrenheit, ϵ the linear coefficient of temperature expansion, l the length of the horizontal projection of the half arch, and $(\delta_h)' = 2.152 \times 10^{-3}$, then the change in thrust due to temperature change t is:

$$H_t = \frac{\epsilon\, t\, l}{(\delta_h)'} \qquad (34)$$

Table 8(*b*) gives, in skeleton form, the computations (from sheet ED9) for calculating the crown displacements, horizontal and vertical, for each degree of change of temperature. Note that Cols. 3 and 9, Table 8(*b*), apply also to erection stages I to IX, inclusive.

From Eq. 32*a*, the moment at the centroid due to final erection loads (ties removed and jacks in place at the crown) was found to be 122,760 kip-ft. From

TABLE 7.—DISPLACEMENTS DUE TO A UNIT HORIZONTAL FORCE OF ONE

Point	(*a*) RIB PROPERTIES					(*b*) BENDING				
	(1)	(2)	(3)	(4)	(5)	(18)	(19)	(20)	(21)	(22)
									(20) ×(3)	
	L	$I \times 10^{-3}$	$\frac{L}{I} \times 10^3$	$\sin \alpha$	$\cos \alpha$	Y	$(Y - 44.82)$ M	M'' and M'	$6E\tau$ $\times 10^3$	$6E\theta$ $\times 10^3$
	Ft	In²-ft²	1/(in²-ft)			Ft	Kip-ft		Kips per	
SB						150.00	105.18	....		55.9
								310.46	55.9	
	9.25	51.26	0.180	0.5497	0.8354					
								305.38	55.0	
SP13						144.92	100.10			282.3
								291.08	227.3	
	17.30	22.14	0.781	0.5331	0.8461					
								281.86	220.1	
(etc.)										
....										
....										
....										
								−133.16	−513.6	
SP1						0.00	−44.82			513.6
								....	....	ϕ_c =

Eqs. 32*b*, 33, and 34, the thrust at the centroid due to these same loads, at 60° (which is 10° above the normal or camber temperature of 50°, previously stated) was found to be 2,719 + 76 + 15 = 2,810 kips. These are the thrust and moment which, if applied at the elastic centroid of the half rib, would make it possible to join the two adjacent half ribs. Therefore, to determine the corresponding moment and thrust at the crown of the closed arch, it was only necessary to transfer them to that point, using the distance $Y_c = \frac{\delta_h}{\phi_c} = 44.8$ ft as the lever arm for H. The nominal moment and thrust at the crown of the closed arch as erected at 60° were found to be: $M = 122{,}760 - 2{,}810 \times 44.8 = 3{,}130$ kip-ft; and $H = 2{,}810$ kips.

Since one pair of the jacks at the crown, which were to be used for weighing the thrust and moment, were located above the rib and one pair below, on 13.92-ft centers, the proportion of this load to be taken by each pair of jacks (at 60° F) was:

$$\text{Top jack load, } + \frac{3{,}130}{13.92} + \frac{2{,}810}{2} = 1{,}630 \text{ kips}$$

Kip (See Fig. 27(*a*)) Applied at the Elastic Centroid (Half Arch)

Deformation						(*c*) Thrust Deformation					Point
(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	
Σ(22)	(23) ×(1)	(24) ×(4)	(24) ×(5)	Σ(25)	Σ(26)		(29) ×(1)		(30) ×10³ ÷(31)	(32) ×(5)	
$6E\phi \times 10^3$	$6Eu \times 10^3$	$6Eu \times 10^3 \times \sin\alpha$	$6Eu \times 10^3 \times \cos\alpha$	$6E\delta_h \times 10^3$ ←	$6E\delta_v \times 10^3$ ↑	P	PL	A	$PL \times 10^3 \div A$	$PL \times 10^3 \div A \times \cos\alpha$	
sq in.	Kip-ft per sq in.					Kips	Kip-ft	In.²	Kip-ft per sq in.		
				0	0						SB
55.9	517	284	432								
				284	432						SP13
338.2	5,851	3,119	4,951								
											(etc.)
											....
											....
											....
											SP1
0											

TABLE 7.—

Point	(c)—(*Continued*)			(d) Shear Deformation						
	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)
	(32) ×(4)	Σ(33)	Σ(34)		(37) ×(1)	Web area	(38) $\times 10^3$ ÷(39)	(40) ×(4)	(40) ×(5)	Σ(41)
	$\frac{PL}{}\times 10^3 \div A \times \sin\alpha$	$E\,\delta_h \times 10^3$ ←	$E\,\delta_v \times 10^3$ ↓	V	$V\,L$	A_w	$V\,L \times 10^3 \div A_w$	$V\,L \times 10^3 \div A_w \times \sin\alpha$	$V\,L \times 10^3 \div A_w \times \cos\alpha$	$E \times \delta_h \times 10^3 \div 2.6$
	Kip-ft per sq in.			Kips	Kip-ft	In.^{2}	Kip-ft per sq.in.			
SB		0	0							0
(etc.)										
....										
....										
....										
SP1		728.48	209.86							215.5

and

$$\text{Bottom jack load, } -\frac{3{,}130}{13.92}+\frac{2{,}810}{2} = 1{,}180 \text{ kips}$$

Closure Computations

Since there was no assurance in advance that it would be possible to survey the abutments and fabricate and erect the arch without any appreciable errors, it had been decided to fabricate the ribs with a normal opening of 11 in. at the crown, and, after weighing in the proper crown thrust and moment, to measure this actual opening and fabricate a piece that would fit it exactly. This closing piece, referred to as the "keystone piece," was prefabricated 13 in. long on its center line, with its two ends not milled. Provision was made that it could be finally as small as 9 in. on the center line, a variation of ±2 in. from the 11-in. normal.

Prior to closure work in the field, the flexibility of the arch rib itself was investigated completely so that it would be possible to determine the proper correctional movements at the crown to produce theoretical thrust and moment after a weighing had been made.

To determine the effect of unit movements at the crown on the thrust and moment, the results of the elastic centroid calculation were used. Thus, the

(Continued)

(44)	(45)	(46)	(47)	(48)	(49)	(50)	(51)	(52)	Point
	(e) Summary of Displacements, in Thousandths of a Foot								
$\Sigma(42)$	$\frac{(27)}{6E}$	$\frac{(35)}{E}$	$\frac{(43)\times 2.6}{E}$	Σ	$\frac{(28)}{6E}$	$\frac{(36)}{E}$	$\frac{(44)\times 2.6}{E}$	Σ	
$E\times\delta_v \times 10^3 \div 2.6$	Horizontal $= (\delta_h)' \times 10^3$				Vertical $= (\delta_v)' \times 10^3$				
	Bending ←	Thrust ←	Shear ←	Total ←	Bending ↑	Thrust ↓	Shear ↑	Total ↑	
0	0	0	0	0	0	0	0	0	SB
									(etc.)
									....
									....
									....
537.64	2.1084	0.0248	0.0191	2.1523	6.5566	0.0071	0.0475	6.5970	SP1

horizontal force required at the centroid to deflect the centroid (and the crown) of the half rib a distance of $\frac{1}{2}$ in. (0.0417 ft) horizontally without rotation was found to be, since $(\delta_h)' = 2.152 \times 10^{-3}$, $H = \dfrac{0.0417}{2.152 \times 10^{-3}} = 20$ kips. Similarly, the moment on the half rib, which would rotate the crown of the rib through an angle whose tangent was $\frac{1}{2}$ in. in $\dfrac{12.23 \text{ ft}}{2}$, or $0.0417 \times \dfrac{2}{12.23}$, was (from Eq. 31a): $M = 0.0417 \times \dfrac{2}{12.23} \times \dfrac{1}{0.000001263} = 5{,}400$ kip-ft. This moment, applied at the centroid, would cause only rotation at the centroid, but would produce deflection at the crown. The horizontal component of this deflection would be (from Eq. 31b): $\delta_h = 5{,}400 \times 0.0000566$ ft; and, since for this condition only rotation was wanted at the crown, it was necessary to introduce a restoring force (away from the skewback) at the centroid to eliminate this horizontal deflection at the crown. This restoring force was $H = M \times \dfrac{\delta_h}{(\delta_h)'} = 5{,}400 \times \dfrac{0.0000566}{0.002152} = 142$ kips.

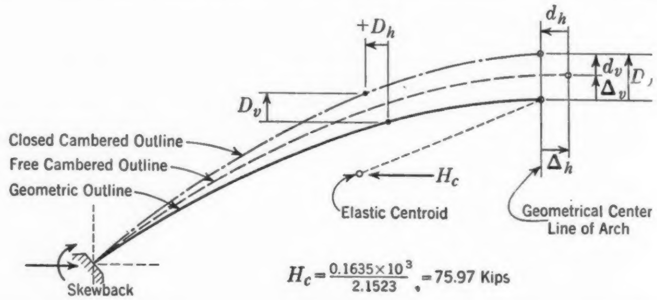
There were thus two basic conditions—one to produce a $\frac{1}{2}$-in. translation of the crown without rotation, or a 1-in. opening between the two ribs; and the other to produce a rotation at the crown, without translation, of such magnitude

that the opening at the top jack would be increased by 1 in., and that at the bottom jack would be decreased by 1 in. This is shown graphically in Fig. 28.

From these basic values, combinations were made which enabled the fabricators to determine readily the expected movement for a given change in jack load, or the expected change in jack load for a given movement. It was obvious

TABLE 8.—DISPLACEMENT (IN FEET) DUE TO

(*a*) DISPLACEMENTS DUE TO CAMBER



$$H_c = \frac{0.1635 \times 10^3}{2.1523} = 75.97 \text{ Kips}$$

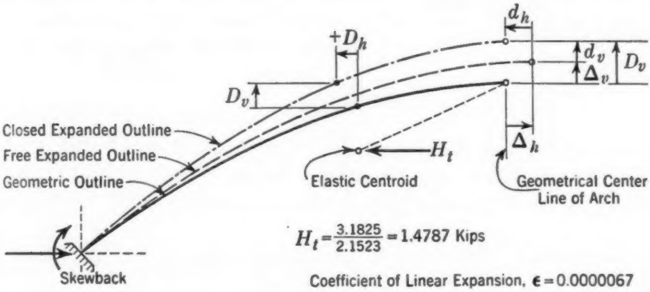
Point	Horizontal Component							Vertical Component					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
	Camber (sheet ED2) ∂l	cos α	$\partial l \times \cos\alpha$	$\Sigma\partial l \times \cos\alpha$	Table 7, Col. 48	$\delta_h \times 75.97$	$d_h - \Delta_h$	sin α	$\partial l \times \sin\alpha$	$\Sigma\partial l \times \sin\alpha$	Table 7, Col. 52	$\delta_v \times 75.97$	$d_v+\Delta_v$
				Δ_h →	$\delta_h \times 10^3$ ←	d_h ←	D_h + ← − →			Δ_v ↑	$\delta_v \times 10^3$ ↑	d_v ↑	D_v ↑
SB				0	0	0	0			0	0	0	0
	0.0015	0.8354	0.0013					0.5497	0.0008				
SP13				0.0013	0.0020	0.0002	−0.0011			0.0008	0.0027	0.0002	0.0010
	0.0076	0.8461	0.0064					0.5331	0.0041				
SP12				0.0077	0.0472	0.0036	−0.0041			0.0049	0.0729	0.0055	0.0104
....	0.0131	0.8654	0.0113					0.5011	0.0066				
....													
....													
TP3				0.1249	2.1691	0.1648	0.0399			0.0472	5.8300	0.4429	0.4901
	0.0080	0.9901	0.0079					0.1401	0.0011				
SP3				0.1328	2.1100	0.1603	0.0275			0.0483	6.0845	0.4622	0.5105
	0.0154	0.9963	0.0153					0.0864	0.0013				
SP2				0.1481	2.1456	0.1630	0.0149			0.0496	6.4654	0.4912	0.5408
	0.0154	0.9996	0.0154					0.0289	0.0004				
SP1				0.1635	2.1523	0.1635	0.0000			0.0500	6.5970	0.5012	0.5512

from the values in the first three columns of Table 9 that the crown could be translated easily, whereas those in the last three columns show that it could be rotated only with great difficulty. This indicated that it would be a relatively simple matter to attain the proper relationship between the load on the top and bottom jacks—that is, to locate the thrust line vertically on the rib—but that it would be difficult to change greatly the total thrust on the two sets of jacks. Thus, there was little control over thrust, and better control over moment, at the crown.

When the foregoing effects of unit jacking had been calculated, it was a matter of simple geometry to construct diagrams tying the movements to the aforementioned criterion—that the upper ends of spandrel columns 1 and 1′ should be at such a distance as to accommodate the prefabricated panel of floor. It appeared that, because of the extreme flexibility of the half ribs in regard to

CAMBER AND TEMPERATURE CHANGE

(b) DISPLACEMENTS FOR EACH DEGREE (F) OF TEMPERATURE INCREASE



Horizontal Component						Vertical Component						Point
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	
X	$475 - X$	$\epsilon x \times 10^3$	Table 7, Col. 48	$\delta_h \times 1.4787 \times 10^3$	$(d_h - \Delta_h) \times 10^3$	Y	$150 - Y$	$\epsilon y \times 10^3$	Table 7, Col. 52	$\delta_v \times 1.4787 \times 10^3$	$(d_v + \Delta_v) \times 10^3$	
	From SB x	$\Delta_h \times 10^3$ →	$\delta_h \times 10^3$ ←	$d_h \times 10^3$ ←	$D_h \times 10^3$ + ← − →		From SB y	$\Delta_v \times 10^3$ ↑	$\delta_v \times 10^3$ ↑	$d_v \times 10^3$ ↑	$D_v \times 10^3$ ↑	
475.00	0	0	0	0	0	150.00	0	0	0	0	0	SB
467.27	7.73	0.0518	0.0020	0.0030	−0.049	144.92	5.08	0.0340	0.0027	0.0040	0.038	SP13
442.70	32.30	0.2164	0.0472	0.0698	−0.147	129.44	20.56	0.1378	0.0729	0.1078	0.246	SP12
												....
												....
110.92	**364.08**	**2.4393**	**2.1691**	**3.2074**	**0.768**	**0.86**	**149.14**	**0.9992**	**5.8300**	**8.6208**	**9.620**	**TP3**
90.00	385.00	2.5795	2.1100	3.1201	0.541	5.20	144.80	0.9702	6.0845	8.9972	9.967	SP3
45.08	429.92	2.8805	2.1456	3.1727	0.292	1.30	148.70	0.9963	6.4654	9.5604	10.557	SP2
0.00	475.00	3.1825	2.1523	3.1826	0.000	0.00	150.00	1.0050	6.5970	9.7550	10.760	SP1

translation, it would not be practicable to change greatly the total crown thrust, and that something quite close to the thrust, as weighed with nominal 11-in. opening, would have to be accepted. Because of the apparently high stresses read on the tieback strands as erection proceeded, there arose some nervousness regarding what the weighed thrust might prove to be. An increase in thrust of 100 kips would increase the computed skewback moment by 15,000 kip-ft. This would be of little import if it arose from a uniformly distributed underestimate of the vertical loads, which would produce counter moments; it could

be a serious matter if it arose from some other inexplicable cause, and was progressively additive from crown to skewback. The contractor did not fear that this latter hypothesis would be true, and was unwilling to agree that, if the total thrust should read higher than computed, it would be obligatory to

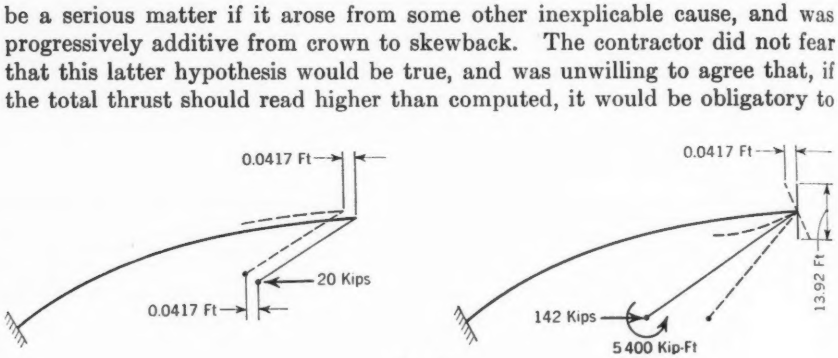


FIG. 28

reduce it. Fortunately, the computed and the weighed total thrust agreed within 1%; the circumstances are recounted because they brought into full relief the necessity, in closing a similarly indeterminate structure, for maintaining the highest possible accuracy in computing the amount and distribution of all dead loads.

TABLE 9.—EXPECTED MOVEMENT, IN INCHES, FOR A GIVEN CHANGE IN JACK LOAD (KIPS)

(+ = Increase, and − = Decrease)

Changes in:	CHANGE IN TOP JACK LOAD (KIPS):					
	+100	0	+50	+314	−366	−52
Bottom jack load (kips)	0	+100	+50	−366	+438	+72
Top jack opening (inches)	+13.2	+11.0	+12.1	+1	0	+1
Bottom jack opening (inches)	+11.2	+9.6	+10.4	0	+1	+1

FIELD CLOSURE

Actual weighing of thrust was done on specially constructed "weighing capsules," one of which was inserted in series with each hydraulic jack. They were sealed, high-pressure hydraulic mechanisms, in which extremely small movements of a plunger under load application were transmitted to a Bourdon tube gage by the enclosed liquid. Each capsule was calibrated with its gage to a load of 400 tons, on the 800,000-lb testing machine at Lehigh University in Bethlehem, Pa. The several gage reading-load curves, as plotted, were straight lines, and were extended to 500 tons, the jack capacity.

Check readings of crown thrusts were also made on 10-in. gages on the hydraulic jack feed lines, and these agreed very closely with the readings on the capsules, although neither the jacks nor the gages had been specially calibrated. The accuracy of hydraulic jacks for reading loads has been questioned many times, but experience on this structure indicates that, if the jacks are in good

condition and are calibrated with the gages, and if readings are always taken on ascending loads, they will read the actual loads with only mirror errors.

A fuller discussion, with record of individual readings on all capsule gages and all jack gages, was published in 1941.[19] Crown thrusts were read after midnight; temperature readings at several points indicated quite stable temperatures between 60° and 64° F. Readings were taken under four different top and bottom crown opening combinations, and it was interesting to see how closely the relationships between change of opening and change of thrust agreed with the results of the intricate computations herein explained.

No better crown opening was found, all factors considered, than the originally selected nominal value of 11 in. The total thrusts at that opening, corrected to 60° F, were as follows, in kips:

Rib	Sum of two top jacks	Sum of two bottom jacks	Total
North rib	1,716	1,123	2,839
South rib	1,757	1,081	2,838
Theoretical	1,630	1,180	2,810

The total measured thrust was therefore only 1% in excess of the computed value; the point of application, however, was above that indicated by computations. The dead weights which seem most open to question are those of the erection travelers and appurtenances, all located near the crown, where they would operate to raise the thrust line. It appears highly probable that not only the thrusts, but also the moments, throughout the structure are within negligible fractions of their computed values. If an excess moment does exist at the crown, it is quite small, does not increase at other points, and is a small proportion of the total moment, including that from partial live load, at any rib section.

Conclusion

If a long, flexible arch bridge of the hingeless type is built with meticulous accuracy in surveying, in fabrication, and in erection, it can be closed on its properly cambered form without need for final adjustments or fear of considerable error in the locked-up stresses from the loads in place at time of closure. If not laid out and built with such meticulous accuracy, it is not probable that such adjustments as can be made at the time of closure would provide sufficient remedies or create full confidence in the stress distribution.

APPENDIX II

For the benefit of any who may wish to make a further and more detailed study of points raised in this paper, the following descriptive index of the information on file at Engineering Societies Library has been prepared.

[19] "Weighing Reactions of Rainbow Bridge, Niagara Falls," by Jonathan Jones, *Civil Engineering*, December, 1941, p. 730.

Sheets	Information
ED1	Basic geometry.
ED2	Camber calculations and shop working points.
ED3	Areas, bending properties, and capacities of rib sections and splices.
ED4 to ED8, inclusive	The vertical and horizontal displacements of splice points, tie points, and crown resulting from unit vertical or horizontal forces acting successively at the tie points (see Fig. 24).
ED9	The location of the elastic centroid of the half arch; the vertical and horizontal displacements, for the half arch, of splice points, tie points, and crown, resulting from unit horizontal thrust or unit moment acting at the elastic centroid; these same displacements, for the closed arch, resulting from camber and from change in temperature (see Tables 6, 7, and 8).
ED10 to ED15, inclusive	The vertical and horizontal displacements of splice points, tie points, and crown under erection dead loads, for the various erection stages, considering the erected portion of the arch to be cantilevered from the skewback (see Table 5).
ED16	The resisting moment of the anchor bolts at the skewback.
ED17	The theoretical crown thrust and moment in the closed arch under erection loads; the horizontal and vertical displacements of splice points, tie points TP6 and TP3, and crown under these loads; the moments and thrusts at the splice points for these loads. The erection loads used were based on carefully made preliminary estimates, but were found to be in error by small amounts when final shop bills were completed. The final calculations for theoretical thrust and moment at the crown were made from influence lines constructed from this basic information (see sheets A129, and A137 to A140, inclusive).
ED18	Tabulation of field information for arch elevations, strand stresses, and strand adjustments; index to calculation (A) sheets which are supplementary to these ED sheets.
ED19 and ED20	Wind stresses and displacements during erection.
ED21 to ED23, inclusive	Backstay strand setting and tension charts.

For index to calculation (A) sheets, and their relationship to the foregoing ED sheets, see sheet ED18. The following calculation sheets referred to on

sheet ED18 are included with the information on file in the Engineering Societies Library:

A19 to A24, inclusive, with charts; A30 to A135, inclusive.

The following subsequent sheets, not referred to on sheet ED18, are also included:

A137	Final revision in erection dead loads at time of closure at crown.
A138	Effect of these latter revisions in erection dead load on theoretical thrust and moment at crown.
A139	Final calculation of total theoretical thrust and moment at crown.
A140	Theoretical elevations on top of cover plate at splices, at closure.

ERECTION OF STEEL SUPERSTRUCTURE

By E. L. Durkee,[20] M. Am. Soc. C. E.

Synopsis

The erection of the steel superstructure of the main 950-ft span of the Rainbow Bridge is described in this paper. Since it is the longest, single, fixed arch span in the world unusual care was exercised in planning the method of erection as well as carrying out the actual erection operations in the field.

The paper describes the general method of erection and the erection equipment. The most unusual features are the tieback system of cable supports used to cantilever the two half arches from each side out to the center of the span; the method of adjusting these cable supports and of measuring the stresses in them; the method of surveying and controlling the profile of the arch ribs during the various erection stages; the measurement of the crown opening left for the purpose of determining the crown stresses; and the fabrication of a special keystone section to make the actual stresses agree with the computed ones for the dead load of the arch ribs. Also, the erection of the floor steel and the placing of the concrete deck are described.

The general features and the design of the arch are covered in the first paper of the Symposium. The calculation of the erection stresses in the arch and tieback system used for controlling the fabrication and erection of the arch ribs is contained in the third paper.

Introduction

Award of the general contract for the fabrication and erection of the 950-ft main arch span of the Rainbow Bridge, including the concrete deck, was made to the Bethlehem Steel Company early in May, 1940. In the construction operations it was required that all workmen employed should be residents of the country where such operations were being done. It was desirable, however, that the field work be under one management and it was agreed that all construction operations be under the supervision of Bethlehem.

The entire responsibility for the exact position and elevation of the arch was that of the contractor. He was required to check all control lines and grades, including the measurement of the distance across the river between the skewbacks made by the engineers, as well as the exact angle of the concrete skewbacks. The engineers' triangulation of the main span showed a length of 950.016 ft, and the contractor's check measurement gave 950.013 ft based on entirely independent base lines—a difference of only 1 in 317,000.

[20] Res. Engr., Erection Dept., Bethlehem Steel Co., Alameda, Calif.

Because of the construction program for the concrete approaches, work on the main span had to be started in the middle of December and was carried on under extremely unfavorable working conditions during the next three months. In spite of this, and the difficulty of securing adequate help, work progressed steadily throughout the winter, without any serious delays and without a single fatality or serious accident of any kind. The work was completed within the allotted contract time and the bridge was opened to traffic on November 1, 1941.

Method of Erection

Cantilever Tieback System.—The arch, obviously, could not be erected on falsework. Although a single bent might have been placed on the steep and rocky shore on either side of the river, it would have been too close to the abutments to have been of much use. To construct bents farther out in the river was impossible because of the sharp angle of the rocky bottom which sloped at about 45° to a maximum depth estimated at 175 ft. Since construction was to be undertaken in the winter and spring, there was also the same danger from ice such as destroyed the Honeymoon span (which the Rainbow Bridge replaced).

An overhead tieback system of support was the obvious solution. Since the arch ribs are entirely self-supporting (there being no spandrel truss system), it was decided to cantilever only the arch ribs and their bracing, which would reduce the dead load to be supported during erection considerably and also permit temporary use of the spandrel columns and floor material in the overhead falsework. This method of erection is not new and has been used in its essential principles in several other arch spans, notably the Hell Gate arch in New York, N. Y., the Sydney Harbour arch in Sydney, Australia, and the Croton Lake arch in Westchester County, New York.

It was found that the first three panels of arch rib, from the abutment to point 10, could be cantilevered on the thirty-two 3-in. diameter upset anchor bolts in each abutment, thus locating the first point of support at point 10. Three other points, at panels 8, 6, and 3, were chosen at which to connect the forward cable ties so as to keep the stresses in these ties and the connection details within reasonable limits considering the weight to be supported and the angle of inclination of the ties. Fig. 29 shows a general view of one half the span cantilevered out to the center with the supporting cable ties and other erection equipment used.

Cable Bents.—To support these ties, a steel tower or bent 130 ft high and 56 ft 2 in. between centers of columns was located on the end of the concrete approach. This height was such as to give a favorable angle of inclination to the forward ties at panel point 3 and keep the stress in, and number of, these ties within reasonable limits. Each leg of the tower was composed of two of the permanent spandrel columns, stitch-bolted together along one face, all riveting in this face being omitted in the shop except for enough rivets with countersunk heads to hold the enclosed box sections together. Each individual column was about 3 ft square, giving a bent column 3 ft by 6 ft. A special base and top section was fabricated to provide for a hinged bearing at the base and suitable details at the top to which to connect the forward ties and backstays.

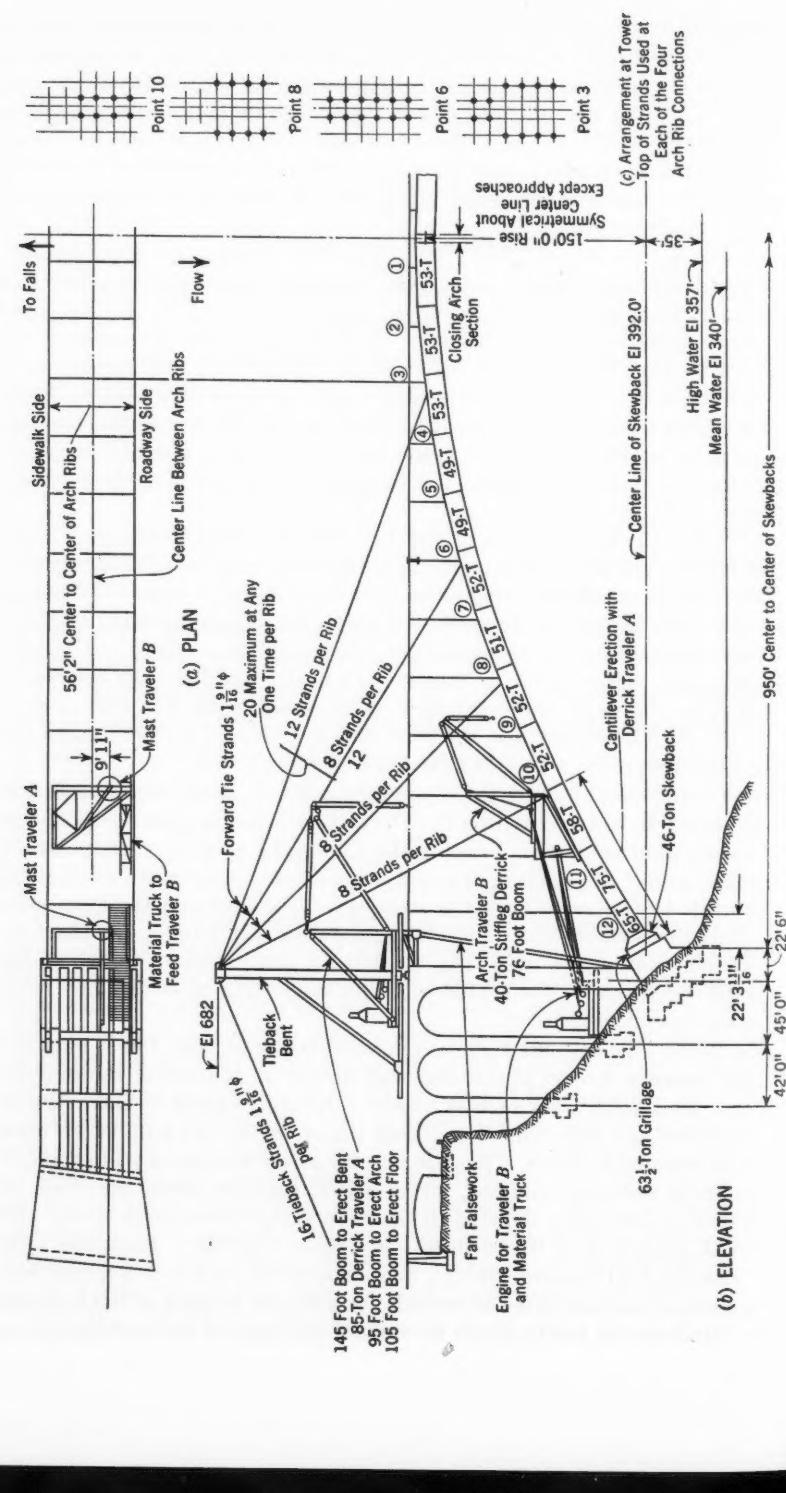


FIG. 29.—VIEW OF CANTILEVER TIEBACK SYSTEM

The top of each tower column consisted of four gusset plates of hammer-head shape, two of $1\frac{3}{4}$-in. thickness and two of $1\frac{3}{8}$-in. thickness, each providing for the connection of four (on the two outside plates) or six (on the two inside plates) forward ties through short, heavy, pin-connected links. Fig. 30 shows the general make-up of this bent, and Table 10 indicates the reactions.

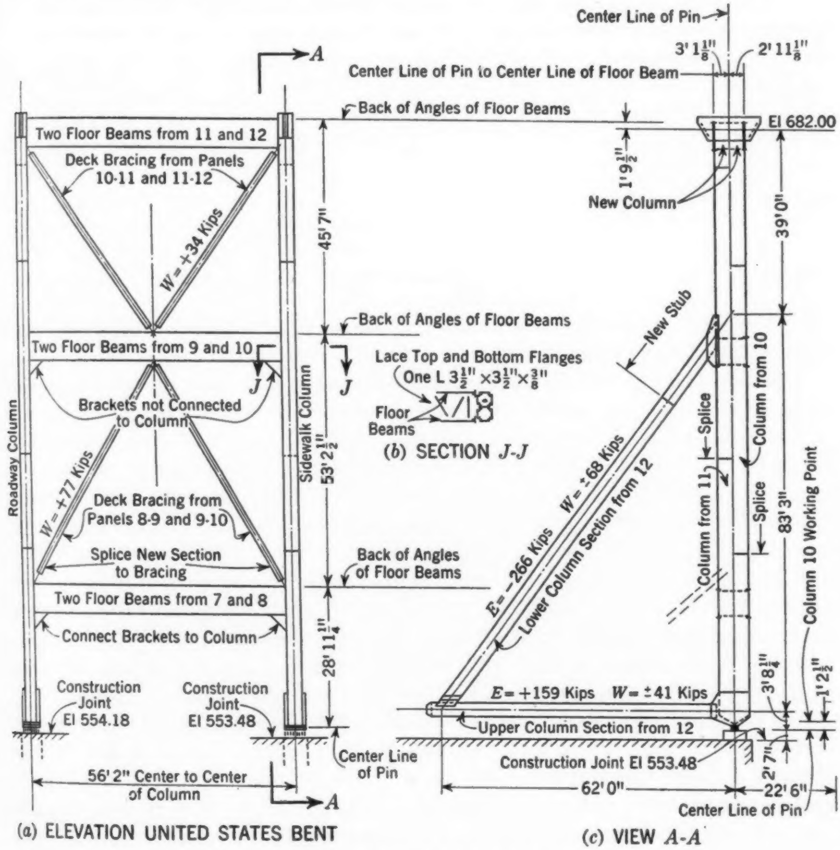


Fig. 30.—Make-Up of Cable Bents

The columns were braced in their narrow direction by three struts each made up of a pair of floor beams from the permanent bridge deck, and each pair of which was braced laterally with temporary angle latticing. The two panels of diagonal bracing completing this bent consisted of permanent lateral bracing also from the deck. An open, unbraced panel of 24 ft was left at the base to permit the passage of the arch-rib sections and other material.

At the top and on each side of each column was a double-deck, steel-framed cage providing two working platforms (see Fig. 31) each 3.5 ft wide by 30.5 ft long. These provided complete access to the various groups of cable

strands in the backstays and forward ties both for erecting them and later for measuring the stresses in these strands with the special cable dynamometer. On top of each column was a small, steel jinniwink or A-frame derrick with a single sheave at the peak. This rig was detailed with a central, vertical pin

TABLE 10.—REACTIONS IN CABLE BENTS (SEE FIG. 30), IN KIPS

Description	*E*	*W*	Total
Maximum Reactions:			
Under stiffener	+213	±55[a]	+268
Under column	+1,799	±65[b]	+1,864
Minimum Reaction:			
Under column	+97	−55[a]	+42
Maximum Horizontal Shear:			
Longitudinal, at stiffener	+85	±45[a]	+130
Transverse, at column	....	±27[b]	....

[a] Longitudinal. [b] Transverse.

about which it could be swung, enabling the rig to face shoreward for the erection of the backstay cables and riverward for erecting the forward ties.

Anchorages.—The top of the cable bent was guyed back on the shore side by means of backstay cables connected to concrete anchorage blocks embedded in the solid rock, there being a separate anchorage for each column. Their location, 360 ft and 375 ft in back of the cable bent, was such as to give the same inclination to the backstays on each side of the river. On the Canadian side of the river, the bare rock surface is exposed, as there is no overburden near the rim of the gorge. Here the anchorage blocks were embedded 6 ft to 8 ft in the solid limestone and extended 17 ft to 20 ft above the rock surface. On the New York side of the river, there is an overburden of 18 ft to 20 ft of clay, gravel, and hardpan which had to be excavated down to the rock surface before blasting out the rock itself. It was also necessary to excavate a trench in front of each anchorage block for the passage of the backstay cables.

In plan, each anchorage consisted essentially of a rectangular block of concrete approximately 16 ft by 19 ft, and 26 ft in height, weighing 550 tons or about 1.5 times the vertical component of the maximum pull of 920 tons in each set of backstay cables (see Fig. 32). The front face of each anchorage in the embedded rock was inclined so as to be normal to the resultant of the cable pull and the weight of the concrete, this resultant thrust being downward at an angle of approximately 12°. In the lower half of each anchorage was embedded a set of structural bars projecting just above the rock surface for connecting the links and bars designed to take the pull of the tiebacks. The embedded anchorage bars spread out fanwise so as to distribute their load throughout the entire area of the anchorage.

In Fig. 33 members AF are sets of three angles and members AB and LK are sets of eight bars.

In excavating the north anchorage pit on the west side of the river, the rock at the surface on the front was found to be of inferior quality and several large uncemented areas between the natural bed planes were encountered during the

excavation. Rock excavation for the approach structures in front of this anchorage also was extended somewhat farther than expected and was cut up further with both transverse and longitudinal sewer trenches. The anchorages, therefore, were excavated 2 ft deeper than originally intended in the north pit and 1 ft deeper in the south pit to insure that the reaction from the backstays would be carried safely into the rock.

In order to prove the value of the rock and the safety of the anchorage design in general, a jacking test was devised for the north anchorage pit on the Canadian side. The entire front face of the excavation was timbered, and grout was poured between the timbering and the rock to give a uniform bearing. Three hydraulic jacks of 350-ton and 500-ton capacity, laid horizontally in the pit, were used to apply a maximum horizontal thrust of 900 tons to 1,000 tons to simulate the inclined design reaction of 870 tons. The thrust from these

FIG. 31.—VIEW OF CABLE BENT, WITH WORKING PLATFORMS AND JINNIWINKS

jacks was taken by timbering against the rear face of the pit in which a shelf had to be excavated as shown in Fig. 34. Observation points were established on the rock surface in front of the pit and compared with other points set in the rock well to either side of the excavation so as to detect any possible movement. It was intended to apply the test load for at least 12 hours, but weather conditions made this impossible. For the 5 or 6 hours during which load actually was applied, no movement or distress of any kind in the rock in front of or around the anchorage pit could be observed.

As a matter of interest, it may be noted that all anchorage blocks have been left permanently in place. The two on the Canadian side are incorporated in the basement of the approach structure, forming part of the walls. The exterior faces of these anchorages were given the same surface treatment as the other building walls at this level. On the New York side, the anchorage pits

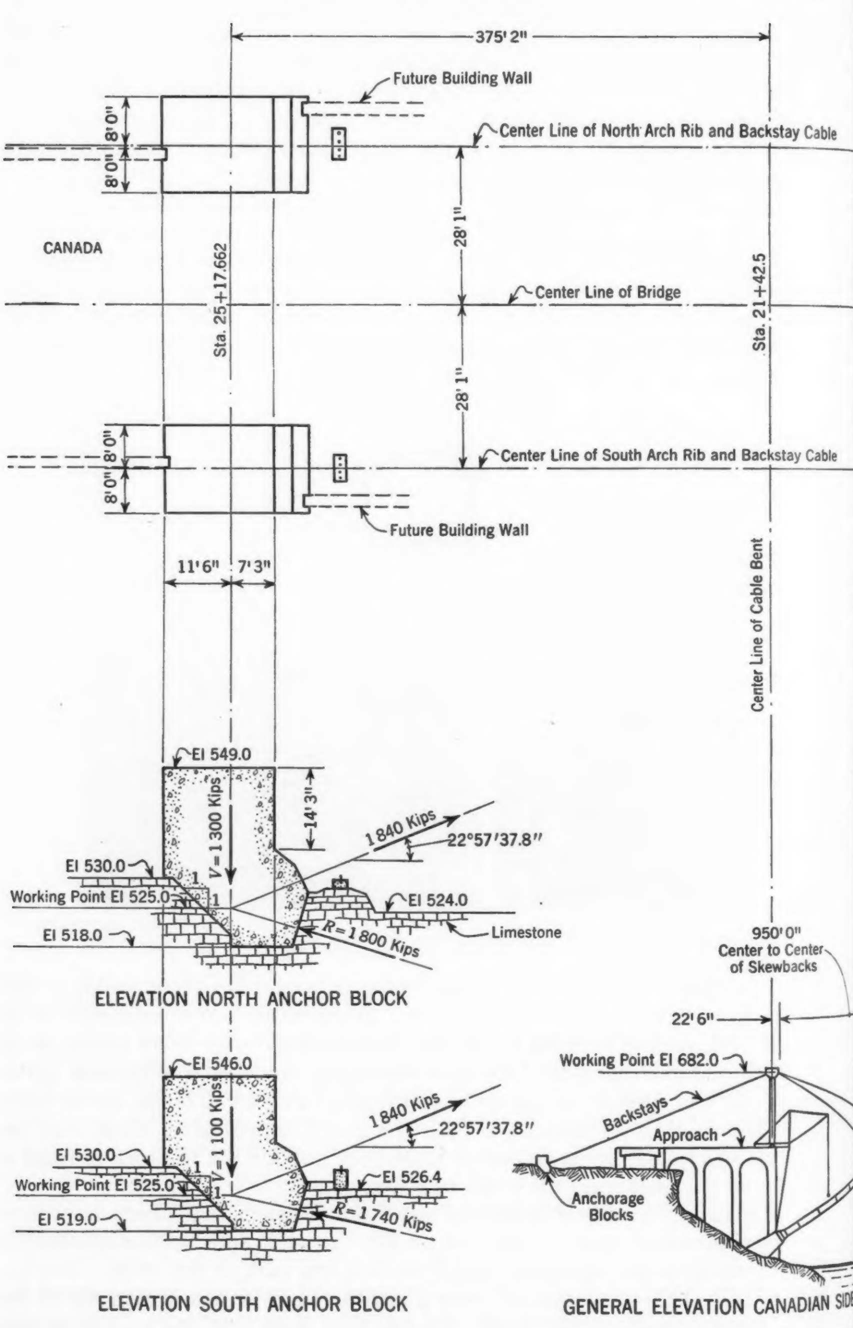

375' 2"
Future Building Wall
Center Line of North Arch Rib and Backstay Cable
8'0"
8'0"
28' 1"
CANADA
Sta. 25+17.662
Center Line of Bridge
Sta. 21+42.5
28' 1"
Center Line of South Arch Rib and Backstay Cable
8'0"
8'0"
Future Building Wall
11'6"
7'3"
Center Line of Cable Bent
El 549.0
14'3"
V=1 300 Kips
1 840 Kips
22°57'37.8"
El 530.0
Working Point El 525.0
El 524.0
El 518.0
R=1 800 Kips
Limestone
950'0" Center to Center of Skewbacks
ELEVATION NORTH ANCHOR BLOCK
22'6"
El 546.0
Working Point El 682.0
V=1 100 Kips
1 840 Kips
22°57'37.8"
Backstays
Approach
El 530.0
El 526.4
Working Point El 525.0
Anchorage Blocks
El 519.0
R=1 740 Kips
ELEVATION SOUTH ANCHOR BLOCK
GENERAL ELEVATION CANADIAN SIDE

FIG. 32.—

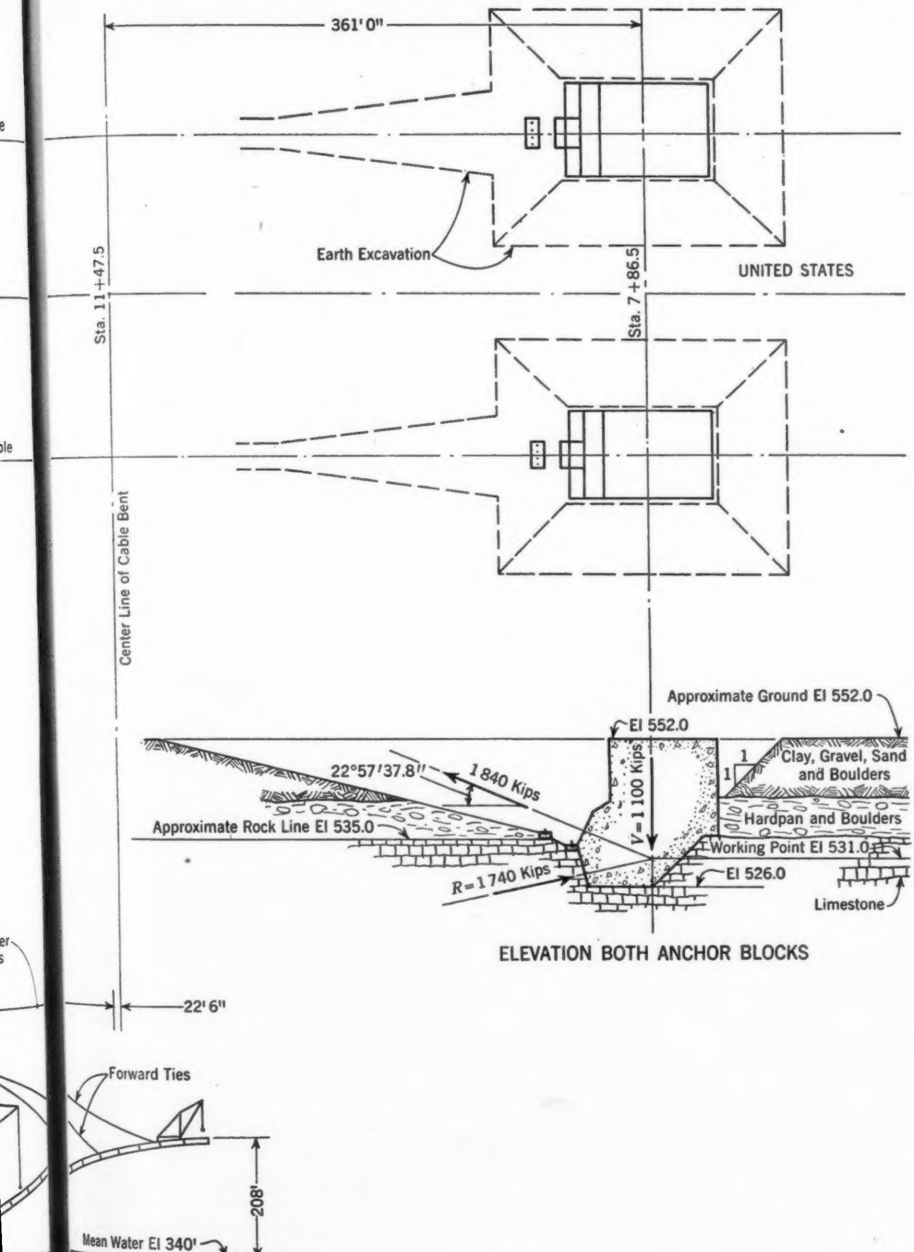


FIG. 32—ANCHORAGE DETAILS

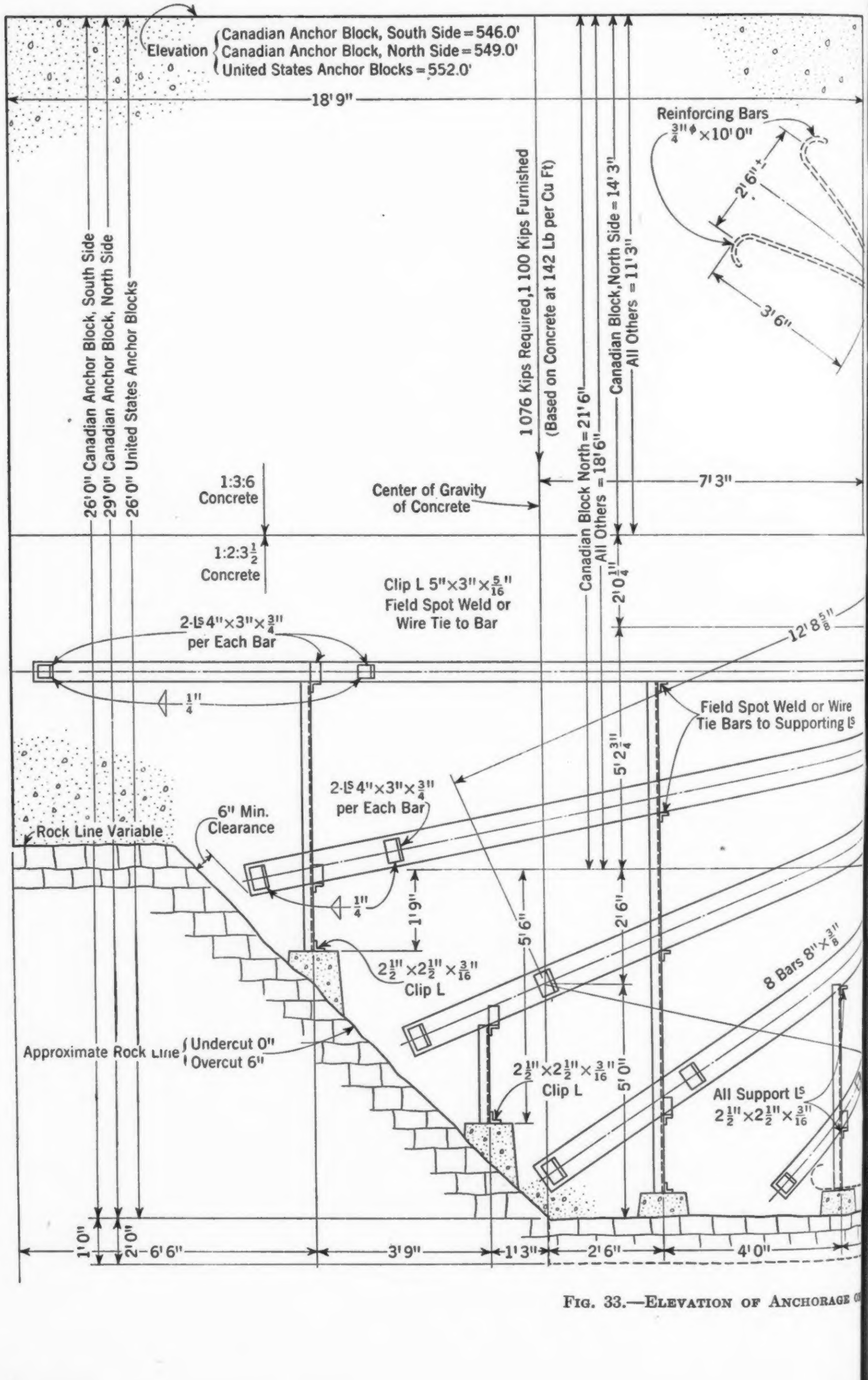
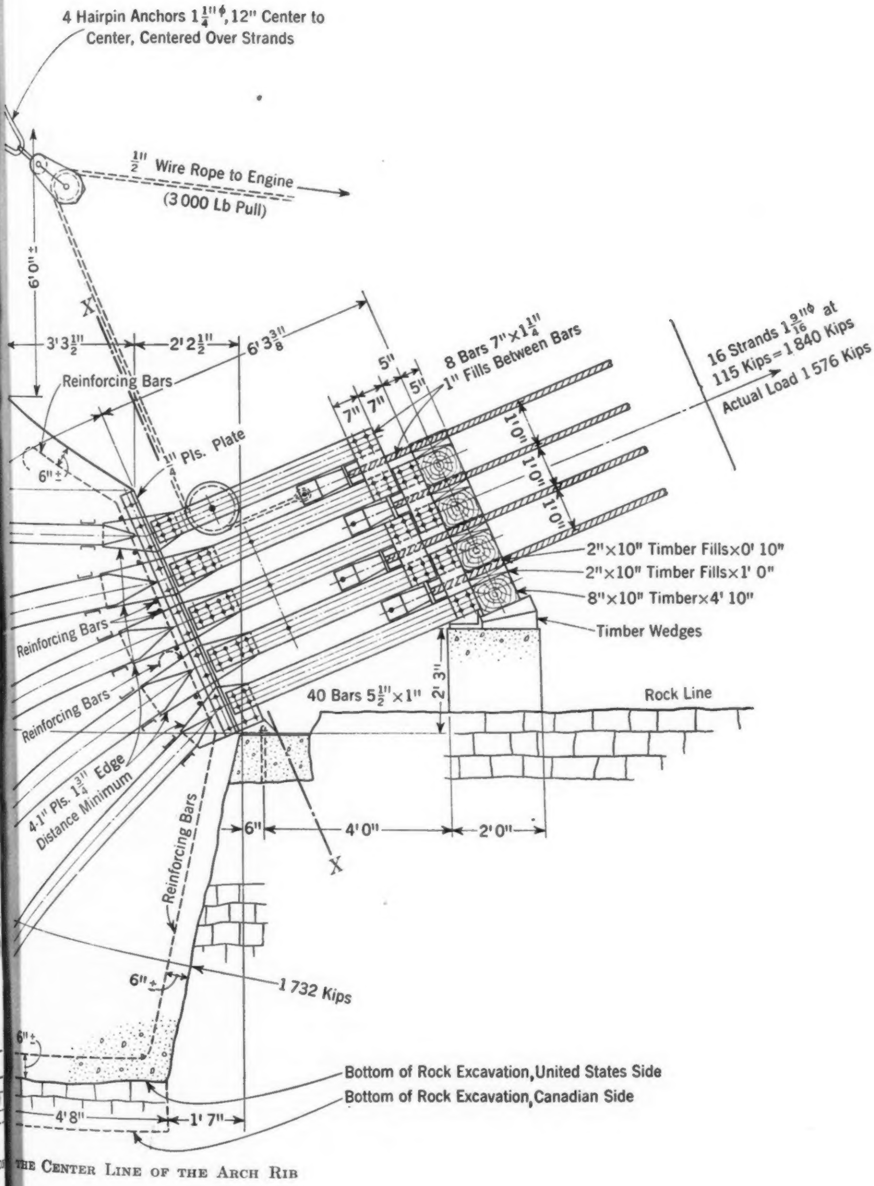


FIG. 33.—ELEVATION OF ANCHORAGE O



...AGE OF THE CENTER LINE OF THE ARCH RIB

were backfilled with earth washed down in place with a stream from a fire hose. The permanent paving in the approach plaza is approximately 2 ft above the top of these two anchorages.

Backstays.—Each of the four sets of tiebacks extending from the top of the cable bent down to the anchorage block consisted of a group of sixteen

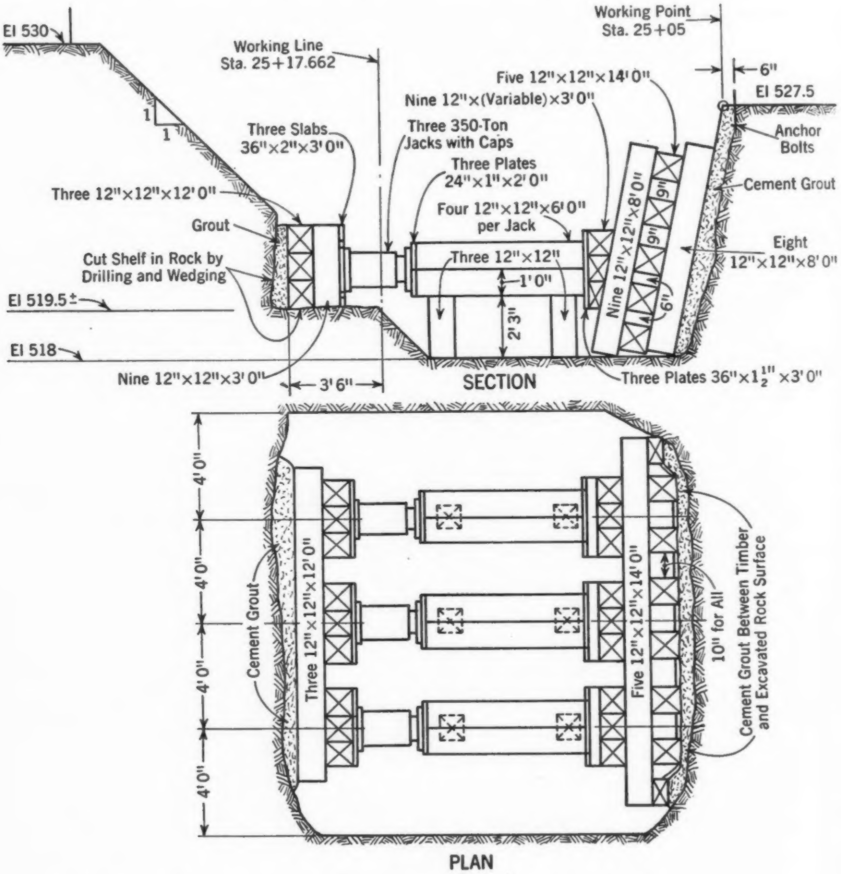


FIG. 34.—JACKING TEST, NORTH ANCHORAGE, CANADIAN SIDE

bridge strands each $1\frac{9}{16}$ in. in diameter with a minimum breaking strength of 155 tons (see Fig. 33). Their normal maximum working stress was 50 tons, increased to 60 tons for wind and ice load. Although they had been prestrained by having been used previously as footwalk cables in the erection of the Golden Gate and Tacoma Narrows suspension bridges, they were prestressed to 150,000 lb for 30 min before being socketed at both ends to carefully calculated lengths under a stress of 100,000 lb. These strands were connected to the tower top through an open socket pin-connected to a short

heavy link, which in turn was pin-connected to a pair of links which were pinned off to the hammer-head gusset plates forming the top of the column. The sockets at the lower end of the strands were of the closed bearing type. They were nested between groups of links bolted to the embedded anchorage steel projecting from the face of the concrete, and the head of the socket bore against pairs of transverse plates bolted to these links. The only adjustment was by means of shim plates between the bearing sockets and these bars (see Fig. 33).

The cable bent was tipped back from its vertical position approximately 3 ft and supported during this time by means of a pair of stifflegs and sills made up of spandrel columns from the permanent structure (see Fig. 30). The rear end of each sill was blocked up from, and tied down to, the concrete approach by means of embedded steel anchors. The actual lean of the tower top was determined by a plumb bob consisting of a steel piano wire and a short section of railroad rail, the latter immersed in a bucket of oil. The lean of 3 ft was designed to give a stress of approximately 5,000 lb in each backstay cable at 60° F. This was well within the pulling capacity of the single runner line used to adjust these strands which first were hung from the top of the tower and the lower end then hauled back into the anchorage connection.

The correct strand stress for the actual tower lean and temperature was insured by surveying the strands for elevation at the quarter point where a wooden tower was located for this purpose. The minimum thickness of the adjusting shim provided at the anchorage was $\frac{1}{4}$ in. which proved to be sufficient to give practically uniform sag to the four cables in each of the four horizontal layers. After this adjustment, no further attempt was made at any time during the erection to survey or to measure the stress in these backstays, entire dependence being placed on the calculations for stress and on measuring and socketing the strands to the calculated lengths.

Forward Ties or Tiebacks.—Two sets of forward ties were used to support the cantilevered arch ribs alternately at points 10, 8, 6, and 3. They were the same size ($1\frac{9}{16}$ in. in diameter) as for the backstays. At point 10, eight strands were used for each arch rib and were hung in two vertical rows of four each from the two center gusset plates at the top of the cable bent. After these ties were adjusted and the rib cantilevered to point 8, the second set of ties, also eight in number, was connected and adjusted. This second group was hung in two rows of four each from the two outside gussets of the cable bent so that, when erected, they passed outside of the first group still in place at point 10.

While the ribs were being cantilevered from point 8 to point 6, the ties at point 10 were disconnected from the rib and slacked back to the abutment where they were left hanging vertically from the top of the tower. An adjustable working platform was provided which could be raised or lowered to bring the workmen up to the lower end of these strands. Lengthening "pennant" strands then were attached to this first group of panel 10 ties to make them long enough for use at panel 6. For this point, four additional strands also were hung from the two inside gussets at the top of the bent in addition

to the eight lengthened strands from point 10, making a total of twelve strands for point 6.

These twelve strands had to be erected by swinging them out through the group supporting the rib at panel 8. When they were finally connected and adjusted, cantilever erection of the arch rib was continued to panel 3 supported on the strands at both panel 8 and panel 6. At point 3 the final group of twelve strands was connected as follows:

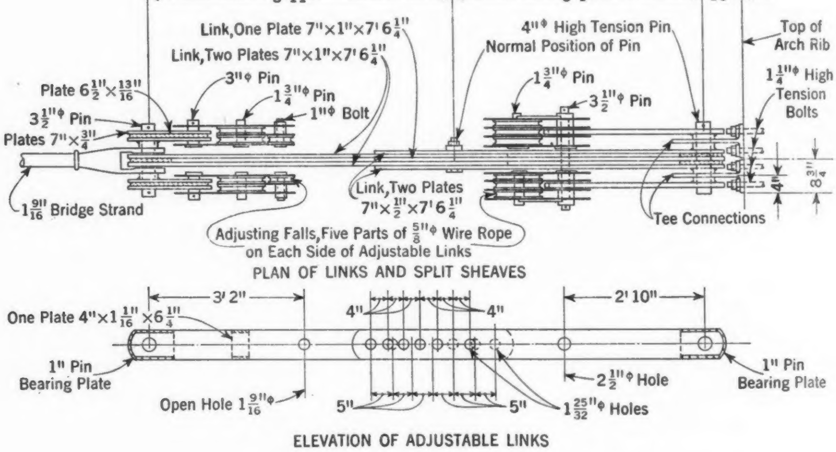


FIG. 35.—ADJUSTABLE LINKS AND SPLIT SHEAVES

The first or uppermost four strands at panel 6 were removed one at a time, slacked back on the arch rib, and lengthened with a pennant. The lengthened strand then was dragged out on the arch rib to panel 3 where it was connected and adjusted. This operation was repeated four times, giving four strands at panel 3, eight at panel 6, and eight still in place at panel 8. These latter then could be disconnected, lengthened, swung out, and connected at panel 3. This gave twelve strands at panel 3 and eight at panel 6, a total of twenty strands per rib, on which cantilever erection was continued to the crown.

The connection of the forward ties to the arch ribs was through a pair of 18-in. heavy T-sections spaced 8 in. on centers and bolted to the top flange of the rib by $1\frac{1}{4}$-in. diameter high tensile bolts. The arch ribs were reinforced inside by diaphragms which took the load from the T-sections and transferred it to the web plates.

The connection of each forward strand to the T-sections was through an adjustable set of links shown in Fig. 35. The lower end of the strand carried an open socket to which was pin-connected the upper link consisting of two plates 7 in. by 1 in. with pinholes in the lower end spaced 5 in. on centers. Meshing with this upper link was the lower link which consisted of three plates, one 7 in. by 1 in. and the other two 7 in. by $\frac{1}{2}$ in. with pinholes in the upper end spaced 4 in. on centers. The lower end of this link was pin-connected directly

to the T-sections. Pinning these two sets of links together through different sets of pinholes gave different over-all lengths of link varying by 1-in. increments from 12 in. shorter to 18 in. longer than the normal length; and by the same or larger increments to maxima of 24 in. shorter to 30 in. longer.

The strand length was adjusted by means of a double set of falls, one on either side of the adjustable links. Each upper, two-sheave block in these falls was supported on the $3\frac{1}{2}$-in. strand socket pin which was extended for this purpose. The lower, three-sheave block was supported on the 4-in. pin connecting the lower link to the T-sections. This pin also was cantilevered out for this purpose. Each set of falls was reeved up with five parts of $\frac{5}{8}$-in. wire rope, the lead lines of which ran directly to the top of the cable bent where they passed over a pair of sheaves and then part way down the bent where they were clamped together to form a bight in which was supported a third or equalizing sheave. This latter sheave was overhauled by a $\frac{7}{8}$-in. wire rope runner line from an engine either on the deck of the approach or on the engine platform. In adjusting a strand, with the adjusting falls in place, a strain first was taken on the equalizing sheave to get the same tension in the two lead lines of the adjusting falls. The two lead lines then were clamped together just above the equalizing sheave so that equal movement was insured in both sets of falls.

The forward ties were erected by a system of horizontal and vertical runner lines, one for each arch rib. The vertical runner line was a $\frac{7}{8}$-in. wire rope run through the jinniwink at the top of the cable bent. The horizontal runner line was also a $\frac{7}{8}$-in. wire rope running from an engine at the abutment out over the arch rib, through a snatch block at the end of the erected steelwork, then back to the abutment where it could be connected to the lower end of the vertical runner line thus forming a "telegraph" system.

The forward ties were shipped on large wooden reels which were unloaded on platforms constructed near the skewbacks. Each strand was unreeled by using the vertical runner line to haul it up to the top of the cable bent where the socket was pinned to the connecting links. To swing a strand out and connect it to the arch rib, the two runner lines were connected through a triangular hitch plate to which the lower end of the strand also was connected. By proper manipulation of the runner lines the strand then was hauled out to the desired point on the rib. Each group of strands was connected at the upper end in two vertical rows whereas the connections at the lower end were in a single row, between the two T-connections. This gave the strands at panel 8 a long, triangular-shaped arrangement with an opening between them of 3 ft at the upper end, tapering to zero near the rib connection.

To erect the strands at panel 6, which had to be swung out through this opening between the strands in place at panel 8, the two runner lines first were hauled up taut so as to bring them above the strands at panel 8. Then they were dropped down through the opening, hooked to the loose strand hanging from the tower top, and pulled back again to a taut position above the panel 8 strands, dragging the loose strand with them. Finally, the runner lines were hauled out to panel 6 where the loose strand was connected and adjusted. The strands at any panel point were removed by a similar use of the runner lines.

Fan Falsework Bent and Platform.—On the river side of the cable bent, and forming an extension to the concrete approach spans, was a steel platform 20 ft long, the outer end of which was supported on two inclined falsework columns themselves supported on a concrete seat just above and in back of the concrete skewbacks for supporting the arch ribs. These two columns, 140 ft long and unsupported for their entire length, each were made from two suitable lengths of the permanent spandrel columns. A special end section (which later was removed and discarded) was detailed, both top and bottom. The upper end of each column was tied back by a steel strut connected to anchor bolts embedded in the concrete approach for the purpose. A floor beam from the permanent structure was framed between the upper ends of the columns. Various other struts and bracing members completed the main platform.

The purpose of this falsework bent and platform was to support a stiffleg derrick for unloading and erecting the steelwork for the arch ribs. This material all was trucked out over the approach and through the cable bent so that the derrick could unload it and lower it into place. A refuge bay 15 ft long was cantilevered out beyond the main fan falsework platform to permit the truck hauling the material to come far enough out so that the center of gravity of the arch ribs would be in front of the cable bent.

Erection Equipment

Yards and Unloading Equipment.—All material for erection in the permanent structure was shipped by railroad to yards on either side of the river. The heaviest pieces to be handled were two 75-ton arch-rib sections (the second rib sections out from the abutment) approximately 53 ft long. Two other arch-rib sections weighed approximately 65 tons and the two steel grillage sections, designed to spread the load of the arch ribs over an area 12 ft by 22 ft on the concrete skewbacks, weighted $63\frac{1}{2}$ tons each, giving a total of six pieces on each side of the river weighing more than 60 tons.

On the Canadian side, an 85-ton stiffleg derrick was set up and operated by a 100-hp, 3-drum steam hoisting engine. Scrap railroad rail was used for counterweight on the end of each sill. This derrick was capable of handling the heaviest, or 75-ton, loads without assistance. It was placed far enough back from the railroad track to permit the trucks and trailers used for hauling to the bridge site to come between it and the track. The material thus could be unloaded directly from cars to the trucking equipment or could be swung around and stored on the ground beside the derrick.

On the New York side it was not possible, in the yard selected (which was long and narrow), to set up a stiffleg derrick of sufficient capacity, or one that could cover the entire yard area. A locomotive crane of 60-ton capacity was provided operating on one track to unload material from the cars which were brought in on a second, parallel track. For the six pieces weighing more than 60 tons it was necessary to bring in a railroad wrecker crane and use both cranes to unload these pieces, one picking each end. Otherwise, the locomotive crane unloaded and loaded all materials. The arch-rib sections could be unloaded and stored alongside the crane track when necessary.

Hauling Equipment and Roadways.—All heavy material was hauled from the railroad yards to the bridge site about a mile distant, by heavy tractors and trailer equipment. All arch-rib sections were loaded on two heavy trailers, one under each end of the rib. A 50-ton, goose-neck type of carry-all trailer was used at the front end and a 100-ton dolly trailer under the rear end. The number and spacing of the wheels in each trailer was such as to spread the load satisfactorily over the pavement. The maximum axle and wheel loads were 24 tons on an eight-wheel axle and 20 tons on a four-wheel axle. Pneumatic tires were used on the carry-all trailer (four wheels) and solid tires on the dolly trailer (eight wheels).

Two separate, heavy-duty tractors were used to haul the trailers, one leading and the other following. The rear tractor was used largely for the purpose of steering, but on the Canadian side of the river it also was necessary to help brake the loads down the steep hill between the yard and the bridge site. The tractors were equipped with air brakes.

On the New York side, the new bridge deck was at the same level as the existing ground surface and the loads could be hauled out over the natural ground directly on to the concrete approach. On the Canadian side, the bridge was approximately 20 ft above the bare rock at the edge of the gorge. A steel ramp 315 ft long with 4% grade was constructed to provide access from the ground surface to the level of the deck. This ramp was made of two-column steel bents spaced from 20 ft to 28 ft on centers with an almost solid floor of 12-in. by 65-lb steel H-beams laid longitudinally to act as stringers. The roadway was formed of 4-in. by 12-in. planks laid transversely across these stringers. A pair of 6-in. by 12-in. timber curbs, spaced 12 ft apart and set 6 in. above the deck, completed the roadway. This ramp was located on the south side of the bridge roadway.

A timber roadway, 12 ft wide between curbs, was provided over the concrete approach on each side of the river. It was located on the south side of the structure, on the Canadian side of the bridge where it formed a continuation of the ramp. On the New York side, the roadway was located on the north side of the approach. This roadway led through the cable bent and out to the fan falsework platform beside the unloading derrick. The 15-ft refuge bay cantilevered beyond the end of the fan falsework columns was designed to support the weight of a tractor only. The remainder of the roadway was designed to carry the heaviest wheel loads. The roadway slab on the concrete approaches had not been poured at this time, as only the main arches, floor beams, and stringers were completed. Hence, the timber roadway was needed over these approach spans.

Unloading Derrick, Traveler A.—For each side of the river, a stiffleg derrick of 85-ton capacity was provided to start the erection and to act as an unloading derrick. It was known as traveler A (see Fig. 29), erected on steel platform beams at the shoreward end of the concrete approach by means of a smaller derrick. In its first position, about 60 ft back from the outer end of the approach, the derrick was provided with a boom 145 ft long by which the first fan falsework columns and platform were erected. Each of these columns weighed 36 tons and was set at a radius of about 65 ft. From this same posi-

tion, the second operation was to erect the 130-ft high cable bent, the heaviest lift in which was the 27-ton top section. After completing the bent, the traveler backed off and the boom was shortened to 95 ft. Then, the traveler was moved forward to the end of the concrete work, standing in the opening of the cable bent from which the lower panel of bracing was omitted and from where it erected the steel grillage and skewback sections of the arch. Finally, the traveler moved forward to its third position standing directly over the fan false-work columns, 27.5 ft in front of the cable bent. In this position, it was used to erect the first three panels of the arch itself and the arch-rib traveler, after which it acted as an unloading derrick feeding material from the deck down to the arch rib. After the arch ribs were completely erected and swung, the boom was lengthened to 105 ft and this derrick then was used to erect the spandrel columns, girders, and floor starting from the abutment and moving out to panel point 9 where it removed rib traveler B (Fig. 29) and completed the erection of the floor. Traveler A was itself dismantled and removed by a tractor crane brought out on the material roadway for this purpose.

The beams and track stringers forming the deck used to support and move this traveler were all stringers from the floor of the permanent structure. The traveler was operated by a 100-hp, 3-drum, steam hoisting engine.

Rib Traveler B.—To continue erection of the arch ribs beyond the reach of traveler A, a second traveler, moving on the arch rib itself, was provided. This consisted of a stiffleg derrick normally of 40-ton capacity but reinforced to enable it to handle the heaviest remaining arch-rib sections weighing about 53 tons. A 76-ft boom was used. The derrick was supported, in plan, on a triangular-shaped steel underframe (see Fig. 36). This underframe consisted of a front cross girder spanning between the two arch ribs and a single longitudinal girder over one arch rib only, the rear end of which was supported on an adjustable vertical leg which could be pinned off to different lengths and thus could keep the deck of the traveler level for the varying angle of the arch rib as the traveler advanced from the abutment toward the crown. The derrick was powered by a 4-drum steam hoisting engine supported on an engine platform located on the steep, talus slope at the bottom of the gorge, just in back of the skewbacks, and occupying the area under the outermost span of the concrete approach. The framework of this platform consisted of steel columns seated on shelves in the concrete footings, and steel beams and girders. A 4-in. plank deck and a light housing to enclose the engine and protect the operator completed the structure.

In working position on the arch this traveler was pinned to brackets bolted to the top flange of the ribs, which took the entire reaction. To move the traveler, it was jacked up, the bracket pins pulled, and the traveler lowered on to three, two-wheel trucks, one at each corner. A move was made for each panel of the arch erected. At the outer end of each rib was bolted a steel frame or cathead which supported a set of four sheaves. An eight-part set of falls was reeved up between these sheaves and a three-sheave block at each side of the front end of the traveler. The lead lines from these two sets of falls ran back to the hoisting engine at the abutment where lines from two of the drums were removed to accommodate them. A complete move of the traveler,

including removal and replacement of the lead lines on the engine, required from 3 to 4 hours.

For the first operation, the rib traveler was used to erect the arch ribs and the bracing between them, advancing from panel point 10 to panel point 2, one panel from the crown. After the erection of the keystone pieces, this

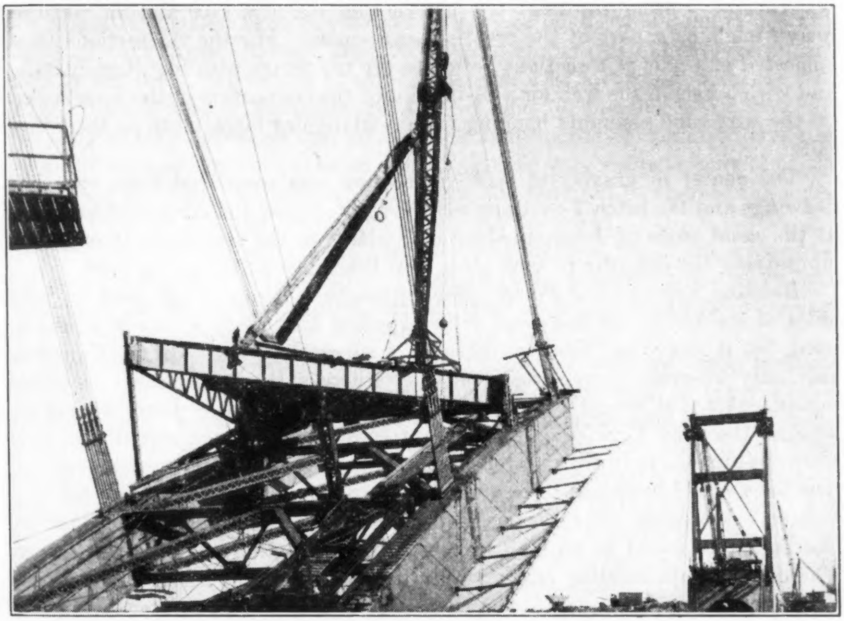


FIG. 36.—VIEW OF RIB TRAVELER B

traveler moved back down the rib to panel point 8, erecting the spandrel columns, girders, and floor framing behind it from the crown to panel point 7. Standing at point 8, it was dismantled and removed by the overhead derrick, which, in the meantime, had been used to erect the floor and had advanced from the abutment out to panel point 9.

Material Truck and Track.—All material was brought within reach of the rib traveler by lowering it from the deck with the unloading derrick A and by landing it on the material truck consisting of a steel framework with a single wheeled truck at each corner. The truck was 38 ft long and had an 8-ft gage between wheels. On one side, it ran on a rail supported over the inside web of the arch rib, the two wheels running on this rail being double-flanged. The two inside wheels ran directly on the top flange of a track beam supported on top of the cross struts at each panel point of the arch rib. These wheels had a plain tread with no flange. The truck was moved by a set of six-part pulling falls, the upper three-sheave block of which was fastened to the right-hand front corner of the rib traveler. The triangular shape of the rib traveler underframe

enabled the material truck to be brought out immediately behind the front cross girder and within easy reach of the derrick. The arch-rib sections were landed on, and securely belted to, special seats on the truck. They were handled one at a time. All other material, such as struts, bracing, and all floor material, was either bolted or securely lashed to the truck and kept from sliding off endwise by means of special brackets or stops.

Hitches.—Special hitches were provided for the arch ribs and for all other heavy loads as a part of the erection equipment. For the rib sections these consisted of a pair of T-sections bolted to the top flange with high-tensile bolts, and with a hole in the web for a $2\frac{3}{4}$-in. pin for the connection of the open socket on the wire rope pennants hanging from a triangular hitch plate on the lifting falls.

The center of gravity of each rib section was computed from the shop drawings and the hitch T-sections were so located that the rib would hang free at the same angle as determined by its location in the structure, thus greatly simplifying the entering of each piece into the splice plates at the joint.

Hoisting Engines and Power.—Steam hoisting engines were used on both sides of the river. It had been fully expected that electric power would be used, but it was found disadvantageous for several reasons. On the Canadian side only 25-cycle power was available, whereas the contractor's electrical equipment was all 60-cycle units. Both 25-cycle and 60-cycle power were available on the New York side. The power companies on each side of the river were not anxious to furnish power, however; their lines already were loaded and new lines would have had to be erected, the entire cost to be borne by the contractor. Moreover, the rates for electric power, taking account of the stand-by charges, were found to be high in comparison with the cost of steam power. Therefore, steam hoisting engines were decided upon and used. Four hoists were required on each side of the river, the boilers being operated by fuel-oil burners.

A 3-drum, 100-hp hoist located on the derrick platform was used to operate traveler A. A similar hoist, but with 4 drums and located on the engine platform at the foot of the gorge, was used to operate the rib traveler B. Three lines operated the derrick boom falls, main load, and runner line, and the fourth line operated the falls used to haul the material truck out along the arch rib. A 3-drum, 50-hp hoist also was placed on the engine platform and operated as a skeleton engine, obtaining its steam from the boiler with the hoist used for traveler B. This hoist was used to operate one set of vertical and horizontal runner lines for one of the arch ribs. The runner lines for the second rib were operated by a fourth hoist of either 2 or 3 drums which was placed on the deck of the concrete approach in back of traveler A. This gave a total of 13 drums in use on each side of the river. In addition, 2 of the 4 drums operating the rib traveler served a double purpose in that they were used for moving the traveler as well as operating the main falls and boom falls of this derrick.

Each derrick boom was swung by a bull wheel operated by a steam swinger engine on the traveler platform. The derrick signalman operated the swinger in each case. For traveler A, steam from the derrick engine boiler was used

for power. Since the engine for traveler B was not with the derrick, but back at the abutment, compressed air was used to operate its swinger engine.

Signal System and Telphones.—Signals between the signalman on the platform of the rib traveler and the hoisting engineer on the engine platform back at the abutment were relayed by an electric light signal system. A signal box was provided at the derrick with two push-button switches for each drum on the engine. A signal light box faced the hoisting engineer and contained two lights for each drum—one red and one green or amber. The lights were operated by dry cells. The green or amber light was used for "going ahead"—that is for raising a load or the boom and the red light for slacking off or lowering. No light at all stopped all operation. Different speeds of operation could be obtained by flashing the lights on and off, giving very delicate control of the load being handled. Considerable pressure was required to operate the push-button switches, making the system practically foolproof.

On traveler A the signalman was always in sight of the hoisting engineer and no signal system was required. Hand signals, relayed as necessary, also were used for the operation of the runner lines, and adjusting falls were used in the erection and adjustment of the forward ties.

Hand-cranked, dry-cell-operated, field telephones were set up in enclosed boxes for use on the work. One telephone was on the deck, at the fan falsework platform, one at the top of each column of the cable bent, and one on the arch rib at traveler B. These telephones were invaluable for the general interchange of information between the foreman (who was usually on the fan falsework platform where he had full view and command of the entire work) and the pushers or subforemen. This was particularly true in the case of the erection and adjustment of the forward-tie cables, the actual adjustment being made on the arch ribs while the stress was being measured at the top of the cable bent.

Compressors and Pneumatic Tools.—Compressed air for the operation of all pneumatic tools and for operating the swinger engine on the rib traveler was furnished by two, 210-cu-ft capacity, fuel-oil compressors set up on the edge of the gorge near the end of the approach spans. Air was piped down the face of the cliff to the abutments and then out the arch ribs. This line also was used to feed air to the work of erecting the center section of the floor, from the crown back to the quarter point. A second air line was run out over the concrete approaches to the deck of the arch span as this was built out from the abutment until it joined the center section, at which time the line on the arch ribs was removed.

In addition to the usual pneumatic riveting hammers, the following pneumatic equipment was used: All bucking up of field rivets was by means of air dollies of various types. Very few rivets are now bucked up by hand dollies. Fitting-up bolts were tightened entirely by the use of pneumatic impact wrenches which have contributed greatly to decreased costs for this work because of their greater speed over hand wrenches and because they exert a greater force, thus insuring tighter work and consequently fewer loose rivets and cutouts.

Safety Nets.—The specifications required the contractor to use safety nets and barriers to safeguard men against falling into the river, these to be left in place until the completion of the work. A type of net similar to that used in the erection of the floor system of the Golden Gate Bridge was used, and, in fact, five nets still on hand from that work were refabricated for use at the Rainbow Bridge. The remainder of the twenty-five nets were newly fabricated.

The average size of the nets was 45 ft by 85 ft (see Fig. 36). They were constructed of $\frac{3}{8}$-in. manila rope laid to form a 6-in. square mesh, one set of ropes being passed through the other at the intersections and being tied with special twine as well. The sides of the net were finished with $\frac{3}{4}$-in. manila ropes extending 12 ft beyond the ends; and at each end there were additional $\frac{3}{4}$-in. tie ropes 8 ft long and spaced 4.5 ft apart with which to fasten the nets to the supports. The edges of adjacent nets were laced together with separate $\frac{3}{4}$-in. manila lines not a part of the nets themselves.

These nets were supported beneath the arch ribs on 6-in. steel H-beams running outside and parallel to the bottom flange of each rib. The beams were held in position 10 ft outside the rib by angle struts and $\frac{1}{2}$-in. wire rope ties extending from the top flange of the rib out to the end of the struts. A diagonal wire rope guy in the plane of the bottom flange of the rib, one per panel, prevented this assembly from swinging downhill due to the slope of the arch. The struts were hinged at their connection to the bottom flange of the ribs so that the entire framework could be assembled to the rib in the yard and folded up alongside it, where it was fastened until after the rib was erected. Each safety net support projected lengthwise about 8 ft beyond the end of the rib toward the center of the span so that when the net was erected, it extended beyond the end of the steelwork and thus provided protection at the splice where the men were connecting the next rib section.

The nets were folded at the factory and shipped in separate bundles. In erecting them, two lines were passed beneath the arch rib from one net support to the other. The net was unfolded to its full width and one end picked by the rib traveler until the entire net hung vertically; then one end of these two lines was connected to the lower end of the net. As the net was lowered over the side, the two lines hauled the lower end under the two ribs and over to the far support. The tie ropes were first secured to the H-beam support on one side, after which the ties on the opposite side were pulled up, using a line from the tugger hoist to give the proper sag in the net, and then tied off. The adjacent edges of the nets were then laced together with a $\frac{3}{4}$-in. manila rope.

Records were kept of all men who fell, or were knocked off the span, into the nets and it is believed that four lives were saved by their use. One man was actually knocked off the rib by a runner line which sagged and then whipped loose, while the other three men either slipped or fell accidentally. Only one man fell from the deck of the bridge, fortunately near the center of the span, and dropped 40 ft. The man knocked off was standing on the top of the arch rib, and the other two fell from scaffolds at the level of the bottom flange of the

arch rib. Fortunately none of these men hit any of the bracing members in their fall and were entirely uninjured. All of them returned to work indicating their confidence in the nets and the job as a whole.

It should be noted that all erection work on the structure, including the concrete deck, was completed without a single fatality or a single serious injury. This is an excellent safety record in view of the locality and the time of year during which the work was undertaken, and the difficult and hazardous nature of the work itself.

Stair Towers, Ladders, Etc.—Access to the engine platforms and skewbacks at the foot of the gorge on each side of the river was by means of a vertical, timber, stair tower approximately 100 ft high down the vertical face of the cliff, followed by a set of steps on the surface of the talus slope. The stair towers were braced from the face of the cliff by struts and guy wires attached to anchors embedded in the rock. These stair towers and steps were constructed by the foundation contractors and then taken over by the superstructure contractor for his work.

Steel ladders were employed to provide access from the deck to the top of the cable bent, running up the inclined stiffleg and then vertically up the column the remaining distance to the platforms at the top of the tower. A steel ladder was provided from the foot part of the way up each fan falsework column to give access to the adjustable working platforms hung at the lower end of the forward ties as they were hanging from the top of the cable bent.

From the skewbacks out to approximately panel point 8, the slope of the arch ribs was so great that a timber ladder arrangement was laid on the top flange of each rib, the cross pieces consisting of 1-in. by 3-in. strips. In addition to this, a 3-ft wide timber walkway with cleats was provided alongside the material track beams. This walkway extended from panel 11 to the rear end of the traveler at panel 2, and enabled the men to keep off the ribs themselves while the forward cable ties were being erected and adjusted.

Short, 12-ft timber ladders were hung over the sides of the arch ribs at various points to give access from the top flange to manholes in the side of the ribs and to the scaffolds hung under the bottom flange at the splices.

At the crown of the arch, opposite the jacking brackets, a timber platform supported on steel beams was constructed, 12 ft wide and 56 ft long, covering the entire distance between the ribs. Large scaffolds were constructed beneath the jacking brackets on the bottom flange of each rib with steel ladders between them and the upper platform. This gave complete and easy access for the erection and adjustment of the jacking equipment and for the measurement of the crown opening during the jacking operations. The upper platform provided a convenient space for the storage of miscellaneous jacking equipment and other tools required at the crown at this time.

Erection of Arch Ribs

General Method of Erection.—Since the 12-ft deep, box-girder, rib sections formed the sole supporting member for the span, it was logical to erect them

only, together with the lateral bracing between them, as the first operation. This reduced by about one half the total weight of steelwork to be supported until the ribs were connected at the crown and made self-supporting. It also permitted the use of the steel not first erected as temporary falsework, traveler track, etc. Approximately 1,000 tons of spandrel columns, girders, and floor beams from the permanent structure were used temporarily for such purposes.

Grillages.—The first piece erected on each concrete skewback was the 63.5-ton steel grillage section made up of plate girders and forming a cellular construction 6 ft deep, 12 ft wide, and 22 ft long. These pieces were designed to distribute the reaction of the arch ribs over the necessary bearing area of concrete. The center third of the top surface of each grillage was milled to a true plane to take the bearing of the steel skewback section, and four corner-bearing surfaces also were finished on the bottom face of each grillage. The original intention was to finish the entire bottom surface to rest directly on the corresponding bearing surface on the concrete skewback itself. This was judged by the contractor to be too difficult of attainment and the following alternative method of securing the proper bearing and alinement was used.

The inclined surface of the concrete was kept about 5 in. below the bottom of the steel grillage except for a 1-ft square pad at each corner. These four pads were to be finished to correct slope and elevation by the foundation contractor. Suitable center lines on each abutment were provided by the engineers and corresponding lines were scribed on the 4-in. slab forming the bottom face of the steel grillage. In order to prove the slope and elevation of these four concrete pads, two triangular-shaped steel frames were built with suitable lateral bracing between them and erected on each abutment, supported on the four concrete pads. They were so designed that, if the pads were finished correctly, the top or horizontal legs of this framework would be in a true horizontal plane, and horizontal and transverse center lines scribed on the steel framework would correspond with the center lines on the concrete work. The original finishing of these concrete pads was neither carefully done nor properly checked and considerable refinishing and adjustment was necessary. As a final check, the milled top bearing surface of each steel grillage also was checked after the grillages had been erected. Instead of a fixed concrete pad, it would have been cheaper and more accurate to provide an embedded, threaded bolt at each corner, with a nut to support and adjust the grillage, and then to survey directly on top of the grillages themselves.

For erection, each grillage was picked up by a special hitch bolted to the top and bottom flange of one of the transverse girder webs. This hitch projected above the grillage and had a pinhole for a pair of short links connecting to the triangular hitch plate on the derrick falls. The hitch was so placed that the grillage hung at approximately its correct inclination in the structure. To guide the grillage in place over the thirty-two 3-in. diameter anchor bolts and to protect the thread on these bolts during this operation, a pilot was provided for each bolt. This consisted of a piece of 4-in. pipe, to one end of which was welded a solid, cone-shaped, steel nose. The pipe simply slipped over the thread on the

anchor bolts and did not screw on, so that the pilots could be placed and removed readily.

After the grillages had been set and adjusted and before erection could continue, it was necessary to fill with grout the 5-in. space under the grillage and to fill with concrete the center, vertical row of cells in the grillage, beneath the steel skewback, to give proper bearing for the thrust of the arch ribs. After erection was completed, the remaining cells were also filled, and the entire grillage surrounded with concrete, thus completely enclosing it and making it a part of the concrete skewback.

Steel Skewbacks.—The second steel member to be erected was the steel skewback section 22 ft deep on the lower side and tapering to the 12-ft depth of the arch rib framing into it on the upper side. This member was a box section, 3 ft wide between the web plates the same as the arch rib, designed to spread the load from the rib over the 22-ft depth of the steel grillage. It was approximately 12 ft in height and weighed about 47 tons. The skewback bolted directly to the steel grillage and in addition was secured by double nuts on the end of each of the thirty-two anchor bolts.

Arch Ribs 13 to 10 and Tiebacks at Panel Point 10.—The three arch-rib sections from the skewback out to panel 10, together with the bracing between them, were erected by cantilevering them out under their own weight, still using the unloading derrick or traveler A to handle them. These three sections weighed, respectively, 65 tons, 75 tons, and 58 tons, each. This resulted in a maximum computed stress in the anchor bolts of not quite 20,000 lb per sq in. (erection stage I).

During this operation, the first set of eight forward-tie ropes had been unreeled and hung from the top of the tower. These were swung forward, using the runner line system and connected to the T-sections at point 10 using the outermost pinholes of the adjustable links so that they still hung slack. After all eight were connected, the adjusting falls were applied to the uppermost strand which was pulled to a stress of approximately 50 tons and pinned off. The falls then were applied to the second strand which was adjusted similarly and which, of course, reduced somewhat the stress in the first strand. The strand stress was measured by the use of a dynamometer, applied at the upper end of the strand. The stress in each successive strand adjusted was reduced slightly in an endeavor to secure approximately equal stress in all eight strands when the final one was adjusted. It was necessary for the total stress in the eight strands to equal as closely as possible the calculated stress of 331,500 lb at 60° F (erection stage II), and also for the individual strand stresses to vary by as little as possible. Three adjustments of the strands were necessary to attain this result. The entire operation of erecting and adjusting this group of strands required 2.5 days.

The sole criterion for adjustment of the arch at this erection stage was the total stress in the eight ties at panel 10 which was calculated to pick up the end of the ribs an amount approximately sufficient to reverse the cantilever stress. Subsequent cantilever erection from points 10 to 8, of course, would increase

the stress in the ties at point 10 and bring the arch ribs down toward their normal position. No survey for profile of the ribs was made at this stage, the measured tie stresses being regarded as critical. The ribs were so stiff that, if they were not adjusted to a correct profile, an excessive stress would have had to be applied in order to bring them to that profile even though the variation at point 10 was only a small fraction of an inch.

With the strands at panel 10 properly adjusted, the rib traveler was erected at point 10, using the overhead derrick for the purpose. The material track and truck also were erected at this time.

Arch Ribs 10 to 8 and Tiebacks at Point 8.—Standing at point 10, the crew on rib traveler B proceeded to erect the next panel of arch ribs and bracing from points 10 to 9 after which it was moved to panel 9; then the crew erected the next panel of the arch from points 9 to 8. This completed erection stage III in which the theoretical total stress in the tiebacks at point 10 was now 493,500 lb at 60° F. No check was made of the strand stresses at this time or of the elevation of the arch rib. The tiebacks for connection at point 8 had been hung in the meantime from the tower and were now swung forward and connected to the T-sections first loosely and then pulled to proper adjustment with the adjusting falls in accordance with the figures furnished by the engineering department for erection stage IIIA.

The total theoretical strand stress at point 8 was now 454,100 lb which reduced the stress in the tiebacks at point 10 to 69,900 lb. This reduced stress permitted the easy removal of the strands at point 10 by the adjusting falls which were used to disconnect them, one at a time, and swing them back to the abutment, still hanging from the tower.

The removal of the strands at point 10 completed erection stage IV and increased the stress in the strands at point 8 to a total of 485,600 lb. No readings of the strand stress were made at this stage. It was assumed that, if the previous adjustments had been made correctly, the stresses at erection stage IV could not be greatly in error. Neither were the arch ribs surveyed for profile at this stage since they were still so stiff that the total strand stress at point 8 was considered to be the proper criterion.

Arch Ribs 8 to 6 and Tiebacks at Point 6.—Arch-rib traveler B was moved immediately to panel point 8 from which position it was used to erect the ribs and bracing from panels 8 to 7, and then moved to panel 7 and erected the ribs from panels 7 to 6. This completed erection stage V, for which the total theoretical stress in the eight tieback strands at point 8 was 772,100 lb. A set of strand readings made at this time for the Canadian side gave a total stress of 786,600 lb for the north rib and 775,900 lb for the south rib, or a variation of less than 2% from the theoretical. The maximum variation in an individual strand stress was considerably higher, one strand being 19% above the average but all others being within 8% of the average. The average variation in strand stress was about 5%. The one highly overstressed strand was slacked off 1 in. as a precaution, but no other adjustment was deemed necessary.

On the New York side of the river, the total measured strand stress at this stage was 20% high for the south rib and 12% high for the north rib, with a maximum variation for a single strand of −10% from the average. Although the total measured stress was considerably higher than the computed stress, there was some question as to the accuracy of the dynamometers being used to measure these stresses, and, for a number of reasons, the readings seemed high, probably by at least 10%. Therefore, no adjustment was made.

Meantime the eight strands released from panel 10 and left hanging from the tower had been lengthened with pennant strands and made ready to be swung forward and connected at point 6. Four additional strands also were added to this group, making a total of twelve. Because of the arrangement of the strands at the tower top, where they were connected in four vertical rows, these twelve strands for panel 6, which were in the two inside rows, had to be swung out between those at panel 8 which were in the two outside rows. Each strand was swung forward and loosely connected at point 6 until all twelve strands were in place, after which they were adjusted individually to the proper tension. This gave a total of twenty strands now supporting each rib, twelve at point 6 and eight at point 8, and completed erection stage VI.

The first survey for the actual elevation of the arch ribs was made at this stage. The elevations were obtained for each panel point by means of a 200-ft steel tape with a specially constructed target and weight hung from the lower end, the total weight being such as to give the correct pull in the tape. The transit used for leveling was set up near the skewback on the side to be surveyed and the height of instrument was obtained from a near-by bench mark. Instead of depending upon the level bubble to obtain a level line, a specially constructed level board was set up on the far side of the river and the instrument was adjusted so as to read the same elevation on the far side of the river as given by the bench mark on the near side of the river. Two such level boards were used, fastened to the concrete abutments, and only a very slight adjustment of the level bubble was required to give the same reading on the two boards.

Theoretical elevations were computed for each erection stage from points 6 to 10 and for each panel point in the arch rib, the point chosen being on top of the splice plate over the milled joint. The tape was kept wound on a small reel. To obtain an elevation the tape was unreeled, and the target and attached weight were lowered over the outside face of the rib, through the safety net, holding the tape at the proper point on the rib. When the target had been properly lined in vertically, a reading was made on the tape at the top edge of the splice plate. Adding this reading plus the distance from the end of the tape to the center of the target, to the height of instrument, the elevation of the panel point was obtained. It was found that these surveys were made best early in the morning, between 6 a.m. and 9 a.m., at which time there was a nearly constant temperature in the steelwork and usually a complete absence of wind. Even a light breeze was enough to set the target swinging sufficiently to make accurate readings difficult and a moderate or heavier wind made them

impossible to obtain. In a light wind, however, the target reading usually could be caught at the bottom of its swing and consistent results obtained.

The first survey of the ribs on the Canadian side of the river for erection stage VI gave results nothing short of remarkable. Of the fourteen panel points surveyed, eight agreed exactly with the theoretical elevations, reading to the nearest 0.01 ft. Of the remaining six points, two were within 0.02 ft, two were within 0.04 ft, and the other two varied by 0.05 and 0.07 ft. The point with the latter variation was at the extreme end of the rib, and the error was never explained satisfactorily. It may have been due to an inaccuracy in the reading, to a minor error in the calculations, or possibly to some local variation in the steelwork itself. No adjustment was made.

On the New York side of the river, the first survey for elevation at erection stage VI showed a maximum variation from theoretical elevation of +0.07 ft at panel 10 and −0.09 ft at panel 6 for the south rib, with corresponding figures of +0.06 ft and −0.05 ft for the north rib. The plotted profiles showed the arch to have slightly too great a curvature due apparently to too great a stress in the strands at point 8 and not enough in those at point 6. Three separate sets of strand adjustments and profile surveys had to be made, after which the measured profiles agreed with the theoretical within 0.04-ft maximum variation. The average variation was only 0.02 ft. It was interesting to note that, at this erection stage, the arch rib was sufficiently limber to respond readily to changes in shape caused by variations in the tieback stresses at panels 8 and 6. The measured strand stresses at this time were higher than those computed by 5%, 6%, 11%, and 16% for the four sets of tiebacks.

Arch Ribs 6 to 3 and Tiebacks at Point 3.—Rib traveler B was moved forward successively to panels 6, 5, and 4, and from these positions the crew erected panels 6–5, 5–4, and 4–3. This completed erection stage VII, with both sets of tiebacks still connected at points 8 and 6. No profile surveys were made for this stage, but complete sets of strand stress measurements were made. These indicated that the tieback stresses at point 6 were somewhat higher, and those at point 8 somewhat lower, than the computed stresses. As the individual strand stress was well within the allowable limit, no adjustment was made.

The next step was the connection of the forward ties at panel 3—one of the most critical and difficult steps in the entire erection procedure. The first four strands at point 3 were obtained from the group at point 6 by: Disconnecting them one at a time; slacking them back on the arch rib; removing the adjustable links; lengthening the strand with a pennant; reconnecting the links; and pulling the lengthened strand forward and connecting it at panel 3. Then, the strand was carefully adjusted to its full stress before the next strand was disconnected at point 6, lengthened, and moved forward. The erection and adjustment of the first four strands at panel 3, with eight strands still in place at panels 6 and 8, was necessary before the strands at panel 8 could be released, and completed erection stage VIIA. A complete survey of the strand stresses was made for this stage to make sure that none of the strands was overstressed,

and that the total measured stress was reasonably close to the computed stress.

The remaining eight strands at panel 8 then were disconnected as a group and swung back hanging free from the tower where they were lengthened by pennant strands. This left the arch ribs in erection stage VIIB, supported on four strands at panel 3 and eight strands at panel 6. An entire set of strand stress measurements also was made for this stage. The stress in the strands at point 3 was shown to be somewhat less than the theoretical, and in those at point 6, somewhat greater than the theoretical. As further cantilever erection would decrease the stress in the ties at point 6, and, as their actual stress at this stage was not critical, no adjustment was made.

The eight lengthened strands then were swung forward to panel 3 where they were connected and adjusted, completing erection stage VIII. At this time a complete survey of the arch ribs again was made both for rib profile and strand stresses. Two adjustments were made on each side of the river, before the third and final surveys showed the ribs to be within a maximum variation of $\frac{5}{8}$ in. from the theoretical profile and the measured strand stresses within reasonably close agreement with the computed ones. In general, the average strand stress was found to be from 10% to 20% higher than the computed, with a maximum variation for a single strand of as much as 50% for the strands with low stress, and 25% to 30% for the strands which were highly stressed. In no case, however, was any individual strand stress found to be greater than the specified design limit. The readings are believed to have been high because of errors in the dynamometer and because the readings were taken relatively close to the socketed ends of the strands where they connected to the tower. Experiments showed that the measured stress decreased the farther away from the end of the strand the measurement was made. In addition to this, the angle at which the instrument had to be used also tended to increase the reading as proved by experiment. The effect of curvature in the cables due to the sag from their own weight was eliminated generally by applying the dynamometer from the side—that is, in the vertical plane of the cable—but in some few cases, the instrument had to be rotated in order to clear adjacent strands, thus causing somewhat higher readings. It is believed the measured strand stress was from 10% to 20% higher than the actual stress.

Arch Ribs, Panel 3 to Crown.—With the final adjustment of the strands in erection stage VIII, traveler B was moved forward to panel 3 in which position the crew erected panels 3 to 2. Then it was moved forward to panel 2 and used to complete the erection of the arch ribs and bracing to the crown where the special jacking brackets on the top and bottom flanges were erected. This completed erection stage IX, the last main erection stage before swinging the two half arches one against the other. Because of the careful control of the profile during previous erection stages, the opposite ribs closed in almost perfect alinement and elevation with respect to each other across the 11-in. gap left at the crown. No adjustment whatever in the elevation or alinement was neces-

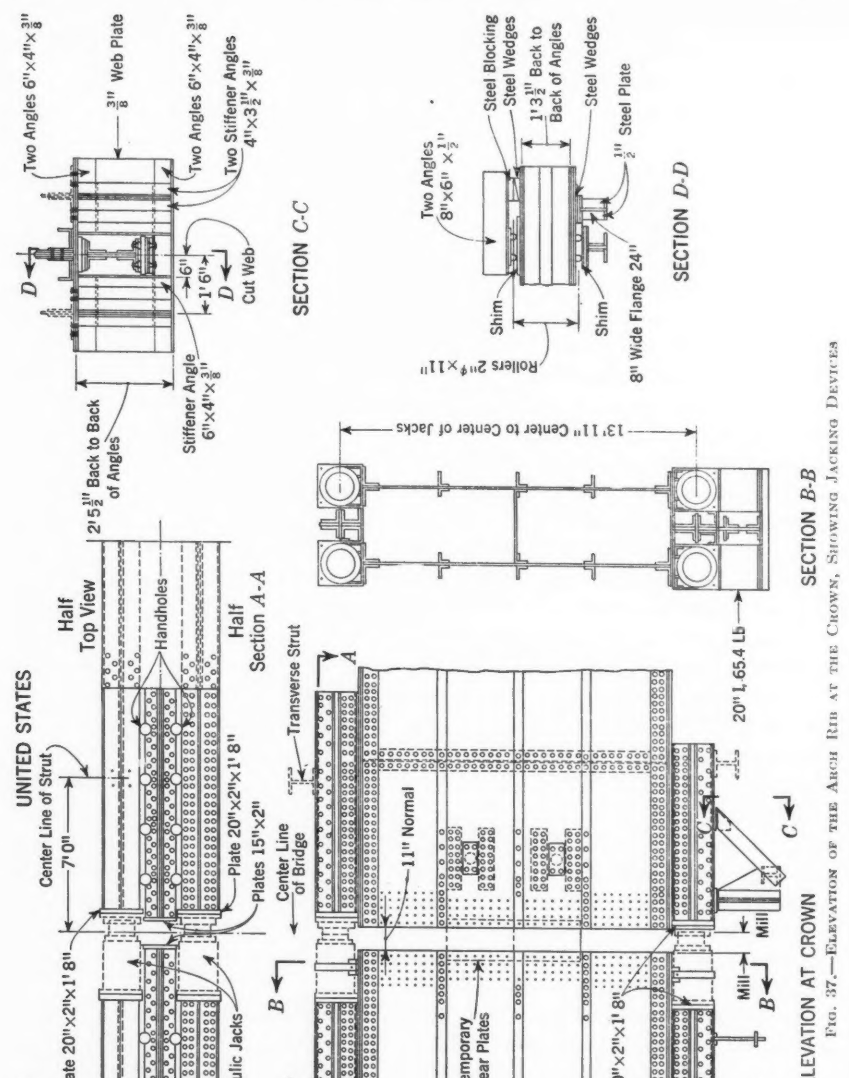


Fig. 37.—Elevation of the Arch Rib at the Crown, Showing Jacking Devices

sary although the ribs were so sensitive at this stage that considerable relative movement was observed from the handling of even light loads by the travelers. Swinging the traveler booms from one side to the other also would produce discernible changes in the elevation. Such relative movements were eliminated by the erection of the special shear devices and temporary lateral bracing erected at the crown for this purpose (see Figs. 37 and 38).

A complete and final survey of the arch ribs was made for erection stage IX, both for rib profile and strand stresses. The actual profile of both ribs, on

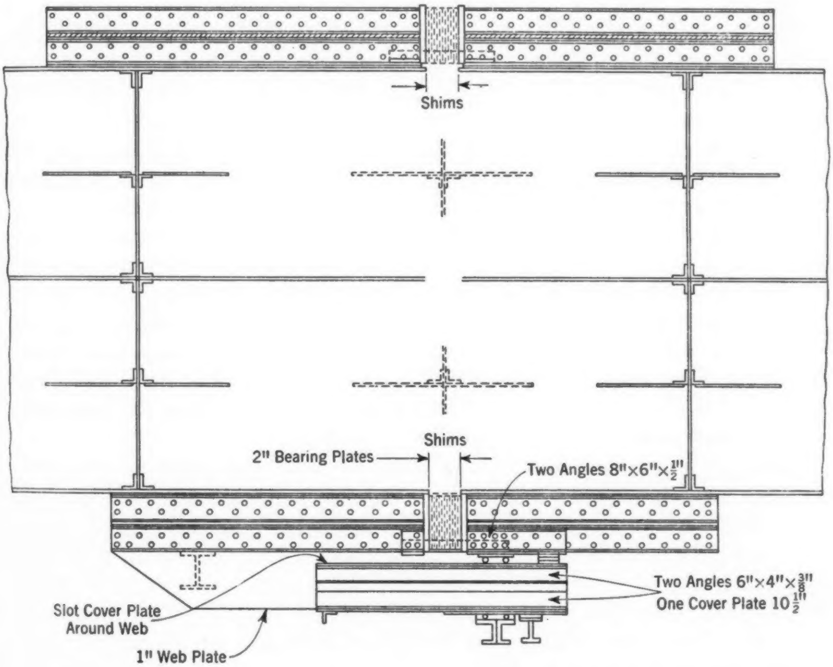


FIG. 38.—SECTION ON THE CENTER LINE OF THE ARCH RIB SHOWING JACKING DEVICES

both sides of the river, was found to be slightly high, by as much as $\frac{5}{8}$ in. to $\frac{3}{4}$ in., between the abutments and panel 7 or 8 where it coincided with the computed profile. From there to the crown it gradually drooped, deflecting to a maximum of 0.16 ft or not quite 2 in. below the computed elevation. The weight of traveler B and the other erection equipment near the crown must have been underestimated. As the cantilevered length was greater than 500 ft and the ribs were only 12 ft deep and supported on only two sets of elastic cables, the structure was extremely flexible in this condition and small loads would cause great changes in profile.

The measured strand stresses at panel 3 were roughly from 30% to 50% higher than the computed stresses, and at panel 6 they varied from 6% too high to 20% too low. Even allowing for the dynamometers giving measured stresses from 10% to 20% too high, they still would be greater than the computed stresses, which bears out the assumption that the erection loads near the crown must have been heavier than estimated.

Fitting Up and Riveting.—Each arch-rib section, when it was erected by cantilevering out from the preceding section, was fitted up in the connecting splice with sufficient pins and bolts to carry the weight of all cantilevered steelwork plus the weight of the rib traveler and other erection equipment. All splices were milled, and the splice plates and rivets were designed to carry only about 60% of the total stress. Computations were made for each joint showing the number of pins and bolts required to carry the erection stresses. In general, sufficient pins were employed to carry the entire erection stress without any dependence on the bolts, which were used mainly to pull the various parts into close contact for riveting. The pins for carrying the erection tension stress were all located in the top-flange splice plates and splice angles. Because of the 12-ft depth of the arch ribs, surprisingly few pins were required to carry this stress.

The cantilever erection stress tended to open the milled joints at the top flange by the amount of play of the pins in the holes, but the bottom flange was brought into tight contact and good bearing. The bottom-flange splice plates and angles were riveted in this condition as soon as possible after the rib was erected. Only a small amount of fitting up was performed in the web splice plates at this time.

After erection had reached the point where a set of forward ties was to be attached and after the ties had been connected and adjusted, the cantilever stress in the two or three forward-rib sections was reversed. This reversal of stress actually put the top flange in compression and thus closed the milled joint. As soon as this occurred and the joint was in good bearing, the top-flange splice plates and angles were fitted up and riveted. The holes in the web splice plates were fitted up and riveted as convenient after both the bottom and top flanges had been riveted.

The lateral bracing between the arch ribs was fitted up as soon as erected but was not riveted immediately. Close watch was kept of the lateral alinement of the arch as cantilever erection progressed, and none of the bracing was riveted until it was certain that the alinement was perfect. Both half arches tended to drift slightly to the south, up to a maximum of about $\frac{5}{8}$ in. to $\frac{3}{4}$ in. A stiff south wind was sufficient to bring the ribs back to alinement, however, and no unusual difficulty was experienced in forcing them back to line by means of temporary cable diagonals tightened with turnbuckles. Riveting of this bracing was kept at least three or four panels behind the erection, however, regardless of the alinement, and the 12-ft deep struts were not riveted until it was determined that the two ribs were within $\frac{3}{8}$ in. of the same elevation.

When the arch ribs were closed at the crown and swung and the last panels of lateral bracing erected and fitted up, the entire arch was found to be in perfect lateral alinement within $\pm \frac{1}{16}$ in. All surveys for lateral alinement were made by means of an engineer's transit set up near the top of the cliff on the center line of the bridge. A foresight was obtained on a bench mark on the far side of the river, and the span lined by means of targets painted on the center of the top cross strut at each panel point.

Closure of Arch Ribs

Jacking Operations at Crown.—The arrangement of the jacking brackets, the hydraulic jacks, and the shims is shown in Figs. 37 and 38. Two 500-ton hydraulic jacks were provided for both the top and the bottom flange for each rib, giving a total jacking capacity of 2,000 tons per rib. The computed rib thrust was about 1,400 tons, of which 800 tons was in the top flange and 600 tons in the bottom flange, due to the dead-load moment at the crown.

As soon as the jacks were fitted up, those at the top flange were operated to increase the top flange opening to 18 in. The bottom-flange jacks were not operated, but the bottom opening automatically increased to approximately 17.5 in. during this procedure and was shimmed snug. During the entire jacking operation, shims were placed in the space provided between the jacking brackets for this purpose, and the load was carried on these shims at all times except during actual jacking movements.

The purpose of this preliminary jacking apart of the ribs was to release the stress, partly, in the forward ties that were still supporting the ribs at panels 6 and 3, particularly at the latter point where the individual strand stresses were about at their maximum and were beyond the capacity of the adjusting falls. The reason for not putting any stress in the bottom flange of the ribs at the crown at this time was that releasing the strands at point 6 would allow the ribs to deflect and thus automatically would induce stress in the bottom flange at the crown, provided the ribs were just in contact at the beginning.

Eight of the twelve strands at point 3 next were completely disconnected, one strand at a time, using the adjusting falls, and the strands were swung back and left hanging from the tower. This caused the cable bent to lean back somewhat and also increased the stress in the remaining four strands at points 3 and 6. In addition, of course, there was an increase in the crown thrust which was taken through the top and bottom shim packs.

At this time, the remaining four strands at panel 3 were left in place to provide the necessary forward pull at the top of the cable bent while the eight strands at panel 6 were removed. Since removal of those strands would increase the stress in these four remaining strands at point 3, the latter first were lengthened by about 30 in. each in small increments so as to avoid overstressing any one strand during this operation. The bottom-flange shims were reduced at this time to $16\frac{7}{8}$ in., thus keeping the bottom-flange load down so that it would not exceed the flange stress that would exist in erection stage X after all cable ties were removed.

Next, the remaining eight strands at panel 6 were completely disconnected, swung back clear of the arch, and left hanging from the tower. Similarly, three of the four remaining strands at panel 3 then were removed and swung back clear of the arch, leaving only one strand per rib still connected at panel 3. In this condition, the top of the cable bent had leaned back very nearly to its original erection condition, about 3 ft out of plumb, thus reducing the stress in the backstay cables so that they readily could be disconnected with a single runner line. Blocking and lashing was placed at the rear end of the sill and stiffleg to support the tower after the last forward ties were removed. The last strand at point 3 then was disconnected, thus completely freeing and swinging the arch ribs which were now entirely self-supporting. The jacks at the crown again were operated and the crown opening reduced from 18 in. in the top and $16\frac{7}{8}$ in. in the bottom to the normal opening of approximately 11 in., top and bottom.

Measurement of Crown Opening.—The 11-in. normal opening at the crown between the arch ribs had been left so that the stresses in the arch ribs might be measured at this point by means of hydraulic jacks and thus compare the actual with the design stresses. The closing keystone piece was prefabricated 13 in. long and provision made so that it could be cut down to a minimum length of 9 in. if circumstances required. By varying the top and bottom lengths of this keystone section, an angular rotation and moment also could be induced at the crown and thus the location of the center of pressure could be changed. A complete description of these calculations is given in the third paper of the Symposium.

In order to compute the crown stresses at this stage of the erection accurately, it was necessary that the dead loads of the erection equipment be known with great accuracy. The weights of the major pieces of erection equipment already had been computed and their effect calculated. All miscellaneous equipment (including drift pins, fitting-up bolts, small tools, timber platforms, and walkways, etc.) was surveyed and its weight computed. Preliminary calculations had shown that the weight on the north rib on the New York side of the river, and on the south rib on the Canadian side, would be somewhat heavier than on the other ribs. It was desirable to have equal weights on the two ribs to avoid two sets of calculations. For this purpose, concrete counterweight blocks weighing 5,000 lb each had been cast and erected at each panel point from points 3 to 10, inclusive, on the south rib on the New York side of the river, and on the north rib on the Canadian side. The traveler booms were swung around to a specified location to produce the desired reaction on the two arch ribs.

In addition to four new 10-in. test gages inserted in the hydraulic pumps, a weighing capsule, which was inserted in line with each jack between the plunger and the jacking bracket, also was provided. These weighing capsules contained an enclosed liquid under initial pressure. They were extremely sensitive to increase of pressure which was measured on accurately calibrated gages. In the actual weighing operations, however, it was found that there

was extremely close agreement in the loads measured both by the weighing capsules and by the ordinary hydraulic gages.

The weighing and measuring operations were performed between midnight and 6 a.m. in order to avoid the effects of temperature changes. The crown opening first was set at its theoretical value of 11 in. $+\frac{3}{32}$ in. at the top flange and $-\frac{3}{32}$ in. at the bottom flange, and the jack stresses were recorded. The crown opening was measured between the milled surfaces at each corner of the flanges by means of machinist's scales, reading to the nearest $\frac{1}{64}$ in. Three other sets of readings for different crown openings also were made in order to check better the relation between the actual and computed stresses and also to determine the most favorable dimensions of the keystone piece. Finally, it was decided that the normal 11-in. opening would produce stresses as close to the computed ones as possibly could be desired. Briefly, the total crown thrust was 1% higher than the computed thrust for each arch rib, although the center of pressure was about 4 in. too high for the north rib and 6.5 in. too high for the south rib. The latter variation was probably due to some inaccuracy in the weight and location of the erection equipment. Since the computed thrust due to dead load of steel must be subject to an error of at least 2% to 3% and since the estimated weight of the erection equipment was undoubtedly in error by at least 5%, it can be appreciated readily that the agreement of the measured and computed thrusts within 1% was nothing short of remarkable. On the other hand, reference to the change in crown opening for various changes in the crown thrust indicates that the two half arch ribs are extremely flexible with regard to translation or change in the crown opening. They are much stiffer with regard to an angular change (that is, a change in moment); hence the conclusion that, for a long flexible hingeless arch fabricated and erected with a high degree of accuracy on accurately surveyed skewbacks, it should not be necessary to provide any closing or keystone piece with the idea of varying the dimensions of such a piece to obtain the computed stresses. Only relatively small changes in the thrust result from relatively large changes in the crown opening.

Keystone Pieces.—The two keystone pieces, one for each arch rib, were an ordinary section of arch rib, 11 in. in length on the neutral axis as finally fabricated. They were slightly wedge-shaped, each face being on a radial line of the rib. The splice plates at the crown were continuous between the two ribs and across the keystone section. All holes in these splice plates were reamed to metal templets in the shop, setting them from the milled ends of the ribs and of the keystone pices as finally finished. There were a total of forty-six splice and filler plates for each keystone piece.

The keystone pieces were erected by swinging them into place sideways into the opening between the arch ribs which was increased to about 11.5 in. to give clearance. A special hitch was provided on the outside of the piece, near the top flange, and a line connected to the inside face at the bottom flange assisted in swinging the piece into place sideways into a vertical position. The splice plates were fitted up to the rib on one side before carefully lowering the

jacks to bring the other side into bearing and correct alinement. No particular difficulty was encountered in this operation, and good holes, together with perfect alinement between the opposing ribs, were obtained.

Removal of Erection Steel.—As soon as the crown opening had been measured and while the keystone pieces were being finished in the fabricating shop, all temporary erection steel used in the tieback system was removed. The forward cable ties hanging from the front face of the tower were disconnected and rereeled. The backstay cables were disconnected from the anchorages, lowered from the top of the tower, and also rereeled. The lower strut and panel of bracing in the cable bent was removed, allowing traveler A to back up to a position about 60 ft behind the tower where the boom again was lengthened to 145 ft. Then, the cable bent, together with the stifflegs and struts used to support it, was dismantled, and all material was returned to the yards for refabricating into permanent columns for use in the final structure. The fan falsework platform and columns also were removed and returned to the yards for refabricating. The traveler boom then was shortened to 95 ft, and the traveler advanced to the end of the concrete approach spans ready to start erection of the floor steel. All of this work was completed by the time the keystone pieces were received and erected.

Erection of Floor Steel

Survey of Arch Ribs and Adjustment of Column Shims.—Immediately after the cable tiebacks had been removed and the crown opening measured, the arch ribs were placed in their normal position with an 11-in. crown opening (the same condition as would exist if the keystone pieces actually were in place). A survey then was run over the arch ribs to determine the relative difference in elevation of like-numbered panel points in the two ribs. The absolute elevation was not desired and was not determined at this time, but it was necessary to know whether one rib was higher than the other so that a correction could be made in order to bring the floor beams level. Difference in elevation was determined by setting up a transit on the arch rib near each panel point and reading the elevation of that point on each rib. This survey showed the north rib to be slightly higher than the south rib throughout its entire length, by a maximum of $\frac{7}{8}$ in. on the Canadian side of the river and $\frac{3}{8}$ in. on the New York side. The difference in elevation at the crown was about $\frac{3}{4}$ in. Both ribs had a very uniform curve or profile from end to end.

The columns had been fabricated with $\frac{1}{2}$ in. of normal shims (made up of one $\frac{1}{4}$-in. shim and two $\frac{1}{8}$-in. shims) between the base plate and the cover plate on the rib. By shifting the proper amount of shims from the north to the south rib, the floor-beam connections at the top of the columns could be brought to the same elevation at any one panel point. On the New York side of the river, a $\frac{1}{8}$-in. shim was removed at each panel point from points 9 to 3, inclusive, on the north rib and added to the south rib. For the three points, 2, 1, and 2, at the crown and for points 9, 10, and 11 on the Canadian side of the river $\frac{1}{4}$ in. of shims was shifted from the north to the south rib. For points 3 to 8, in-

clusive, on the latter side $\frac{3}{8}$ in. of shims was shifted making a relative difference in elevation of $\frac{3}{4}$ in. for the column bases at those points. A $\frac{1}{8}$-in. shim was interchanged at point 12 on the Canadian end of the bridge, and no change was made in points 10, 11, and 12 on the New York end. No explanation is offered for this relatively large difference in elevation between the two ribs which was somewhat surprising after they had followed their theoretical profiles so closely during the various erection stages.

Erection Procedure.—Actual erection of the floor steel was begun as soon as the keystone pieces had been inserted and while they were being fitted up and riveted. At the abutment, traveler A erected the first pair of spandrel columns, which were spliced at the center, for their full height of 142 ft by simply standing them on the arch ribs without any guys or bracing between them. Each column immediately was stayed at the top by erecting the spandrel girder next to the abutment, followed by the floor-beam framing between these columns. These two spandrel girders were supported on sliding seats on the abutment. At this time, they were pulled back several inches to give clearance for erecting the closing floor panel and temporarily anchored in that position to give longitudinal stability to the floor structure being built from the abutment.

At the crown of the arch, travelers B erected the floor steel in front of them, panel by panel, moving backward down the arch rib as each panel was completed. All material for these travelers was fed to them by traveler A and by means of the material truck, as during erection of the arch ribs.

Removal of Travelers; Fitting Up and Riveting.—After traveler A had erected two panels at the abutment, the traveler track was extended and the traveler moved out to panel 11 where it erected two more panels to point 9 and then moved to that position. Traveler B meantime had moved back to panel 7, its final position, from which point its 76-ft boom just could fill in the members in panel 5–6. When this operation was completed, traveler B was dismantled and removed by traveler A which proceeded to fill in the two remaining panels of floor from points 6 to 8.

When the last pair of spandrel girders was erected in panel 6–7, the temporary connection of the two spandrel girders at the abutment was released. These two girders then were jacked forward to close the opening in panel 6–7 and to permit the girders in that panel to be fitted up.

In spite of the fact that there was no sway bracing between the spandrel columns, no difficulty was encountered in keeping them plumb and in holding the entire floor system to correct lateral alinement. This must be attributed very largely to the excellence of the shop work in fabricating the columns and the floor steel.

Fitting up and riveting of the floor steel and bracing followed the erection as closely as possible and was completed within ten days after traveler A had been dismantled and removed by a small tractor crane.

Handrailings.—The two roadway railings and the single sidewalk railing had been erected by the travelers with the other floor steel. With the floor

entirely fitted up and known to be in correct alinement, fitting up of the railing-post connections and alinement of the railing itself were begun. The details of the railing-post connections were such that it was practically necessary to fit them up and rivet them before the concrete deck and sidewalk could be poured. In general the railing-post connections were fabricated with small-sized holes and were fitted up with small bolts until proper alinement was attained. The connections then were reamed out to full size and riveted. The subsequent operation of pouring the concrete deck, and the resultant deflections of the arch rib, produced no noticeable change in the alinement of any of the three railings.

Concrete Deck

The work of forming and pouring the concrete deck and sidewalk slabs was sublet to other contractors, one for the Canadian and one for the New York side. Wooden forms were used throughout, supported on timber joists which were carried in wire hangers resting on top of the stringers.

The main roadway slab reinforcement consisted of prefabricated, welded, bar joists. All longitudinal bracing consisted of plain bars wired to the trusses at the intersections. Ordinary two-way bar reinforcing was employed in the sidewalk slab.

Each contractor used a mixing plant set up near the shore end of the concrete approach. On the Canadian side of the river the mixed concrete was wheeled out over a runway to the center of the bridge where pouring operations were started. On the New York side all concrete was pumped out over the span through an 8-in. pipe line. Concreting was started at the center of the span in one roadway only and proceeded simultaneously in both directions. The sidewalk slab was poured at the same time as the south roadway slab. In order to prevent excessive deflections in the arch ribs, it was specified that not more than four panels of the roadway be poured on one side of the approach than on the other.

The concrete subcontractors also poured the concrete encasing around the steel grillages at the skewbacks, thus completely filling and enclosing these members and making them a part of the concrete abutments.

Painting

Two field coats of aluminum paint were applied to the structure over two coats of proprietary brands of a red lead and iron oxide type of paint.

Application of the first field coat was started before the arch ribs were closed at the crown and continued during the erection of the floor steel and the pouring of the concrete deck. The second field coat was not applied to any exterior surface until the concrete deck immediately above had been poured and the forms removed. All the floor steel received its first coat of paint before the concrete was poured. During concreting operations streams of water were played on the steel to clean it of concrete splash, but, because of an insufficient number of streams and lack of pressure, considerable cleaning had to be done by hand after the concrete had dried before the second field coat could be applied.

Both field coats of paint were applied by brush and no spray painting was done. The aluminum paint used was purchased in the summer of 1940 before its use was prohibited.

Conclusion

Field work was begun on the Canadian side of the river on December 15, 1940, and on the New York side on December 26, 1940. The first permanent steel, consisting of the steel grillage, was erected on the Canadian side on February 14, 1941, and on the New York side on March 6. The last four main-rib sections were erected at the crown on May 19, and the measurement of the crown opening was made on the night of May 26–27. The keystone pieces were received June 11 and erected June 12. Erection of the floor steel was begun on June 13 and completed on July 17. The contract date for starting work on the concrete deck was July 21. The concreting was completed on the Canadian side on September 9 and on the New York side on September 15.

The safety nets were left in place until all concrete work and painting (with the exception of painting the top flanges of the arch ribs) was finished. Removal of the safety nets was completed on October 9, and the final painting was completed on October 24.

The bridge was opened to traffic on November 1, 1941, at which time paving of the approaches and the finishing of the toll booth and quarters for the customs and immigration officials still had not been completed.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CONFORMITY BETWEEN MODEL AND PROTOTYPE A SYMPOSIUM

Discussion

BY MESSRS. JACOB E. WARNOCK AND H. G. DEWEY, JR., MARTIN E. NELSON AND JAMES J. HARTIGAN, J. C. STEVENS AND R. B. COCHRANE, EDWARD SOUCEK, AND FREDERICK R. BROWN

JACOB E. WARNOCK,[36] M. AM. SOC. C. E., AND H. G. DEWEY, JR.,[37] JUN. AM. SOC. C. E.[37a]—It is most gratifying to note the number of excellent discussions provoked by this Symposium. No doubt the wartime duties of many other interested engineers have prevented them from submitting a discussion or concrete evidences of model-prototype conformity. The excellent examples of conformity submitted by Messrs. McConaughy, Hall, and Blaisdell add much to the value of the Symposium.

Professor Streeter has stated that personnel of hydraulic laboratories are inclined to rely upon the model for the answer to all hydraulic questions instead of applying fundamental principles. It is true, of course, that in recent years hydraulic engineers have relied heavily upon hydraulic model studies to solve many unprecedented problems. Professor Streeter knows from his experience that many of these problems are not susceptible to fundamental analysis, and, even if an application of fundamental principles were possible, those responsible for the design of a structure would not be satisfied until a model test had been made. On the other hand, many hydraulic problems have been analyzed in laboratories by application of fundamental principles, resulting in a limited amount of testing or no testing at all. Eventually, hydraulic engineers should become more adept at preparing satisfactory designs of structures by using knowledge gained from previous model tests of

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similar structures and from checks on the prototype performance. As soon as the modern theories of fluid mechanics become more practical and are shown to be entirely adequate, the need for model tests will probably decrease and the general application of fundamental principles should be predominant. Professor Streeter believes that the application of fundamental principles will yield more reliable information at a smaller cost. This might be quite true, provided that the reliability can be established to the satisfaction of all concerned. Unfortunately, this is not generally the case, since some designs so analyzed have either yielded uneconomical solutions or inadequate designs as shown by subsequent model studies. An inspection of some of the older hydraulic structures in the United States also will show this to be true, since model studies were not used so extensively at the time of their design, but fundamental principles were relied upon almost entirely.

Eq. 18, computed by Professor Streeter, is in good agreement with Curve 3, Fig. 13. It will be recalled that Curve 3 was obtained by treating the bellmouth entrance as an orifice, using the pressure drop as measured in the bellmouth of the prototype (Fig. 4). Either Eq. 18 or Curve 3 is more reliable than Curve 4, which was derived from prototype discharge measurements. Since these discharges were obtained by a rather indirect method, they were not used to compute the prototype friction factor.

In regard to neglecting the kinetic energy correction factor, Professor Streeter suggests that, in using $K_e = 1.00$ instead of 1.02, the prototype friction factors in Fig. 6 may be 25% too high. If, in 40 ft of conduit, the measured friction loss is assumed as 2.87 ft for a head of 150 ft, the friction factor would be 0.0073; if $K_e = 1.02$, the friction factor would be 0.00715, or 2% less than the former. Neglecting the kinetic energy correction factor would not change the measured friction loss, of course, but would make the computed friction factors too high (although only about 2% and not 25%). It was previously recognized that the velocity distribution in so short a conduit would not be sufficiently developed to allow a good determination of the friction factor. As Professor Streeter suggests, velocity distributions should have been obtained in the test section, but this was impossible since no provisions were made on the prototype for velocity measurements—another example of the difficulty of obtaining proper measurements and correlation.

In regard to the statement that the discrepancy between model and prototype pressures was partly due to unstable flow in the model, it was felt that viscous effects in the model were more predominant than in the prototype. That this may be true is demonstrated by Mr. Douma (see Fig. 80).

Professor Streeter's demonstration of estimating the prototype pressure drop from crown to invert from the model pressures is particularly interesting. Although the difference in pressure between the crown and invert is of interest, the actual pressures along the crown and invert are of more concern.

Messrs. Tower and Pafford request that hydraulic laboratories compile data from model experiments and prototype measurements in a form suitable for design purposes, thereby aiding designing engineers and reducing the number of model tests of nearly identical structures. This is a good request

and should be given more attention. Unfortunately, those who control the functions of many hydraulic laboratories are primarily interested in the solution of immediate problems and, as yet, have allowed those in the laboratory but little time for compiling data or for conducting research which would eventually furnish design data. In regard to data on the curve of the lower nappe of the sharp-crested weir, the laboratory of the Bureau of Reclamation, in 1942, completed a comprehensive research program on the nappes of such weirs (77).[37b]

Mr. Pafford's suggestion of keeping piezometers open in the prototype by frequent flushing with clear water is recommended, but, as he suggests, it will be a nuisance to the operating personnel. It might be added that it would be more of a nuisance during construction when the danger of plugging is probably the greatest. The use of removable, flexible, corrosion-resisting wire would be excellent for short piezometer tubes, but for long tubing this method would probably be impracticable.

Mr. Gilardi wishes to lift the veil of secrecy surrounding some engineering mistakes, and cites as examples two cases where solutions by model studies were not followed, causing inadequate prototype designs. He seems to blame both the laboratory and project, or consulting engineers. Obviously, if the latter group has a poor working knowledge of model studies and little faith in the recommendations obtained from them, and wishes to adhere strictly to its own preconceived ideas, then unfortunate results may occur. On the other hand, there should be little excuse for well-trained and thoroughly experienced laboratory personnel arriving at erroneous conclusions, provided they receive full instructions relative to the prototype conditions from the project engineers. In this regard it is believed that better results can be obtained to the satisfaction of all concerned if the designing engineers are able to keep in close contact with the laboratory through frequent visits.

Mr. Douma's discussion of similitude is quite appropriate. It is agreed that better results could have been obtained had the model of the outlets been built to a larger scale—the surfaces, however, were about as smooth as could be obtained. Had the tests been run at higher heads, disregarding the scale ratio, it is believed that the pressure analysis would have been more consistent using the dimensionless parameters shown in Figs. 7, 8, 10, and 11. The use of outlet models of sufficient size to obtain a larger Reynolds number so as to be somewhat closer to the prototype values of friction factors would have been impossible because the discharge facilities of the laboratory would have been exceeded. On problems where the prototype Reynolds numbers are not too large, it may be easy to close the gap between model and prototype; generally, however, the value of the Reynolds number in the model would have to be increased considerably to reach the point where the variation between the friction factor and the Reynolds number is negligible and at the same time of the same order of magnitude as the prototype friction factor for the entire range of discharges to be studied.

[37b] Numerals in parentheses, thus: (77), refer to corresponding items in the Bibliography, which appears as the last unit of the Symposium, and at the end of discussion in this issue.

MARTIN E. NELSON,[38] M. AM. SOC. C. E., AND JAMES J. HARTIGAN,[39] ESQ.[39a] —An interesting analysis of the relative effectiveness on lock efficiency of area changes in culverts and ports has been developed by Mr. Kreiss. From Eq. 33, he has derived an expression for the area ratio which satisfies the condition of equal percentage changes in port area and culvert area being equally effective in changing the discharge capacity. When μ is 0.15, this area ratio of 1.45 is numerically about the same as the area ratio for square-cornered ports used in early lock designs.

Also, it can be shown from Eq. 33 that the upper limit of the area ratio, at which percentage increase in port area affects the discharge capacity, is

$$x = \frac{1}{\mu \log_e^{10}} \qquad (37)$$

This limiting area ratio is 2.89 when μ equals 0.15.

For the venturi-shaped ports of the first and third[39b] models, it is obvious from Figs. 25(*a*) and 34(*a*) that the upper limit for effective increase in area ratio has not been reached in either model test. Area ratios for these two models are 2.22 and 1.65, respectively. Similarly, in the third lock prototype, having an area ratio of 2.19, all lock-chamber ports are used effectively. However, in the first lock prototype the distribution of flow curve of Fig. 25(*a*) shows an effective use of only 17 of the 20 ports, corresponding to an area ratio of 1.9. The μ-value satisfying Eq. 37 is 0.23 when the area ratio is 1.9. It would be desirable to have series of tests on venturi-shaped ports, having different divergence angles, to determine whether μ in Eq. 31 remains constant.

Tests have been made on a lock model equipped with ports similar at the culvert end to the bellmouth entrance of the lock-chamber ports of the third lock (Fig. 24), but having parallel instead of divergent walls. This type of port has a larger discharge capacity than the venturi-shaped port for corresponding area ratios. The tests showed that increase in area ratio above 1.8 did not change the discharge capacity of the lock, area ratios of 1.0, 1.4, 1.8, 2.2, and 2.8 having been tested.

The authors are very grateful to Mr. Webster for his description of the testing apparatus used in obtaining performance data in the prototype locks. This apparatus was developed after a considerable investigation of commercial hydraulic testing equipment failed to bring to light any instruments suitably adaptable to the tests in view. The equipment proved to be remarkably well suited to the severe conditions under which it was necessary to operate.

The coordination and timing of tests performed simultaneously and at great distances on the lock chambers were facilitated greatly by using two sets of sending-receiving short-wave radios to transmit signals from the central operating station to the observation stations. For instance, in making simultaneous lock-stage tests at the two ends of a lock, time signals, originated by

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[39b] Corrections for *Transactions:* Changes to be made throughout the paper to distinguish between "first," "second," and "third" models, to conform with Table 3.

an automatic electric timer, were picked up on the radio receiving sets at the observation stations, indicating to the observers when to record the stage on the manometer charts. Likewise, the observers could be kept informed of the start, end, and intermediate progress of the valve operation, and other pertinent information could be transmitted readily from one station to another.

J. C. STEVENS,[40] M. AM. SOC. C. E., AND R. B. COCHRANE,[41] ASSOC. M. AM. SOC. C. E.[41a]—From the standpoint of design of the crest profile, Mr. Grimm's discussion of the writers' paper was enlightening. The writers realized that operation of the dam with the gates in the upstream slots would be abnormal and that such gate operation was for emergency purposes only. Since such operation data were procured with the model, a splendid opportunity to obtain corresponding prototype data was presented. The fact that the prototype pressures obtained with the gate in the upstream slot did not corroborate the model pressures constitutes certain "negative" information which at times is just as valuable as "positive" information. The writers agree most heartily with Mr. Grimm in his statement that "A model study of spillway crests under controlled atmospheric pressure would be timely and of decided value as a research project to supplement present knowledge."

EDWARD SOUCEK,[42] ASSOC. M. AM. SOC. C. E.[42a]—The interest shown in the paper is most gratifying; particular thanks are due to those who have presented previously unpublished data.

Professor Johnson's remarks deal primarily with unsubmerged spillways, but the tests of the Upper Narrows Dam are of much interest. It is regretted that the model details were not illustrated or described. Since the prototype is curved in plan, similarity demands convergent flow over the spillway and radial approach velocity; these conditions are difficult to obtain on a sectional model in a flume only 6 in. wide. End effects sometimes are feared in so narrow a flume, even when a straight spillway section is represented. The agreement between discharge curves of the 1 : 25 and 1 : 40 models seems reassuring in this connection; certainly the agreement tends to confirm the principle that a spillway model need not exceed a reasonable size. The small effect of surface roughness upon the discharge of the two larger models is highly significant. The curve for the 1 : 100 model shows that it is too small for rating the prototype; it may be too small also for accurate determination of the nappe profile. In connection with that phase of the work especially, but also for the discharge measurements, supplementary measurements were undoubtedly necessary to determine that a 1-in. pipe provided complete aeration of the nappe. The writer has used a small U-tube, partly filled with water and connected to the space beneath the nappe, to indicate the subatmospheric and fluctuating pressures that accompany incomplete aeration. This check is recommended strongly as surprisingly large vents have been required in some cases.

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[42a] Received by the Secretary July 1, 1943.

It is doubtful that a "more definite measure of the agreement between the various studies" is gained by use of the exponential equations derived by Professor Johnson. For that purpose, the curves of Fig. 64 leave nothing to be desired. It appears that use of the conventional exponent with a varying coefficient is more convenient for nearly all purposes, cases in which the discharge equation must be integrated being an important exception.

Professor Johnson's statement that "model studies made of existing structures are more likely to be academic in character" is a well-meant generalization; it is questioned only because it is not entirely harmless. In the writer's experience, exceptions have been so frequent that they cannot be said to prove the rule. A few cases will be mentioned: (1) A large spillway was tested to determine a rating which was not otherwise obtainable and for study of the effect of proposed modifications; (2) a large waterway was studied to verify the ability of the model to reproduce wave phenomena previously observed on the prototype—then the model was used not only for the study of proposed modifications but also for investigation of extreme conditions which it was unsafe to create and impracticable to observe on the prototype; and (3) a lock was tested to verify the subsequent use of models for study of proposed structures and for the development of apparatus and technique—an inexpensive, practical, and important modification, which probably will be applied to the existing structure, was discovered as an incidental by-product. It is true that models of existing structures frequently are tested in college laboratories. Such tests may indicate a feasible modification for improved performance or may shed light on a poorly-understood phenomenon which has been observed on the prototype. A useful by-product of the tests described in the paper was their immediate local application for stream gaging under flood conditions. Possibilities for valuable model-prototype comparisons are almost unlimited. A student can make a far worse choice in research problems than a study of an existing structure. Believing that such studies should be encouraged, the writer hopes and believes that their character will be judged by the care with which the subject is selected, the manner in which the experimental work is performed, and the skill with which the test results are interpreted.

Professor Cox points out that a sharp-crested weir is affected by the tailwater elevation when submergence, as defined by the writer, does not exist. The point is well taken; nevertheless, the writer prefers the original definition to the one proposed on the basis that the name of a phenomenon should be descriptive rather than indicative of its effects. The more general term "affected by tailwater" would cover the situation.

The writer does not advocate any significant departure from geometric similarity; accordingly, he agrees with many of the statements by Professor Cox regarding the flow downstream from a submerged dam. He still believes, however, that the tailwater should be measured beyond the effects of the dam whenever possible and that this is possible in most cases. The channels downstream from low dams can be represented reasonably well by a flume, when necessary, by molding the channel bottom to represent the variation in depth with distance from the dam. Only if such representation is impossible would the writer adopt the expedient of measuring the tailwater within the effects of the

dam. In such a case, the problem of relating the observation to the backwater curve downstream from the dam remains unless discharge measurement is the only objective. The error possible in making this estimate is believed at least to equal the effect of a reasonable approximation to the channel cross section in the model. The several advantages of measuring the tailwater beyond the effects of the dam were stated in the paper and need not be reiterated.

Fig. 68 exhibits a tendency similar to that revealed by Fig. 54—the greatest submergence at which the plunging nappe condition can be maintained decreases as the head increases. Professor Cox' correction curve for adjusting the observed submergence would be expected to exhibit a loop or other discontinuity corresponding to the changes in behavior of the nappe. By the use of this curve and the equations which he derived, Professor Cox obtained computed discharges in good agreement with the observations. In at least one other case (50) Professor Cox has used his method with good results. The writer does not know the scope and limitations of the method. Dimensionally, it appears subject to question but it has been used very successfully by Professor Cox.

In stating that no gagings were made when the ice jam was within 500 ft of the dam, the writer was trying to show only that the jam had no effect except the raising of the tailwater to a level somewhat higher than would occur normally at the same range of discharge. It is regretted that this statement led Professor Cox to speculate upon the existence of a reverse bottom current. It is believed that such a current would have been detected from the angle assumed by the cable which supported the meter and the weights. During all except one of the measurements, the distance to the ice jam was much greater. Angularity of subsurface velocities and turbulence may have affected the measurements, producing results similar to those suggested by Professor Cox.

The data presented by Mr. Walton, as he states, are surprising in their close agreement to the U. S. Deep Waterways tests. Probable reasons for this correspondence are considered subsequently. On the basis of Mr. Walton's observations, the writer agrees that the behavior of the nappe requires consideration in some cases. It might be inferred that this effect is present or perceptible only when the spillway is particularly "sensitive" to submergence. The writer's opinions concerning the measurement of heads coincide with those of Mr. Walton; certainly the tailwater may be measured within the region affected by nappe conditions when no alternative exists.

Mr. Tower expresses the now general acceptance of the nappe-shaped profile. It is interesting to point out, however, that the shape of the dam tested was established by an eminent hydraulic engineer, not "by reason of the lack of adequate information" but in order to modify an existing structure most economically, so as to accommodate a large tunnel for steam pipes and other utilities. The dam always is submerged when a major flood is in progress. The writer knows of no evidence that the spillway is not as efficient as a conventional nappe-shaped profile when the submergence is high; there is no deterioration that could be attributed to the shape of the spillway. This excep-

tion to a perfectly valid general rule illustrates the need for complete data before passing judgment on a design.

Professor Mavis presents a useful formula and valuable data. His qualified acceptance of the writer's statements regarding the effect of nappe conditions upon discharge is especially pertinent in view of Mr. Walton's observations. The equation presented is equally applicable to both sections shown in Fig. 73. Since the effect of submergence doubles for each 5% increment in the submergence, the relationship is remembered easily. For a rounded cross section which supports the nappe, the equation is certainly more correct than the widely-quoted and much-used curve based on the U. S. Deep Waterways tests. Further evidence to this effect appears in another discussion which is considered later.

Professor Mavis' conclusion (b) that "the generalized relationship between submergence percentage, p, and the ratio of submerged to free discharge, * * * is valid within the scatter of field observations on the prototype" is accepted but the qualification should not be ignored. For preliminary designs or for estimates of the effect of submergence upon the discharge over well-rounded crests when model data are not available, the equation may be used to good advantage. For analysis of test data and their application to geometrically similar prototypes, retention of the discharge as an independent variable is believed to justify fully the minor complications necessitated.

Messrs. Nelson and Hartigan kindly have devoted a large part of their discussions to the writer's paper. This presentation of new data leaves little opportunity for comment in addition to a statement of approval. Attention is called to the better collocation of those observation points which represent normal tailwater in Fig. 85. These data support the contention that discharge should be treated as an independent variable. The agreement of the "effect of submergence curve" for the well-rounded crest with the equation suggested by Professor Mavis confirms the applicability of the relationship.

Messrs. Nelson and Hartigan measured the water levels beyond the immediate effects of the dam as the writer recommends. The agreement between model data and prototype observation is highly satisfactory, particularly in view of the minor departures from complete similarity. In one respect, these tests were contrary to the writer's findings; the tests did not reveal a consistent relationship between the discharge and the effect of submergence. In this connection, the comments accompanying Figs. 52 and 53 should be emphasized. The tendency for the effect of submergence to increase with the discharge is clearly evident but the consistency of the data is exaggerated to some extent. When the curves were sketched originally, all differed in shape; some actually intersected. By a process of repeated adjustment, all of the curves except the one for the lowest discharge were drawn to conform to an average pattern. It seems possible that a similar smoothing process would reveal a consistent tendency if applied to the data of Messrs. Nelson and Hartigan. Fig. 83 shows an excellent method of representing submerged spillway data graphically.

The last paragraph of Mr. Blaisdell's discussion deals with the writer's use of the discharge as an independent variable; he recommends that this variable

be ignored in determining the effect of submergence. The writer emphatically disagrees with the suggestion and the implied ideas.

At a submergence of 85%, the reduction in discharge produced by submergence may be as little as 7%. The 3% error which Mr. Blaisdell considers tolerable thus amounts to more than 40% of the effect being investigated. For high submergences, the absolute error in discharge can be at least as great as 10%, and much greater if the average curve is based on a small range of discharge. It is true that for submergence exceeding 85% "the submergence curves are steep, and small errors in determining the submergence result in large errors in the submerged discharge correction." However, water levels are observed so easily and accurately that there is no reason whatever for any significant error in the determination of submergence. An average submergence curve is also steep. The remote possibility of a small error in the submergence by no means justifies the deliberate introduction of a large error, particularly when the only possible gain is the avoidance of a simple computation requiring at most a few minutes' time.

The data were presented in a manner intended to expedite comparison with published material. It was stated that a direct method of making the calculation could be derived easily if a large number of calculations were necessary. The data were so arranged that no trial computations were required in order to check the results. The measurements on the model were intended for comparison with prototype measurements; an average difference of a few per cent, probably caused by errors inherent in the method of gaging the prototype, was anticipated and realized. The possibility exists that, in the future, the model measurements will be compared with more accurate prototype observations. Under these conditions, use of Mr. Blaisdell's method would have been improper. If (as Mr. Blaisdell states) the result of a refinement which avoids an error of 10% or more is a fictitious accuracy, literature records many unwarranted model studies.

In the application of submerged spillway data to a particular problem, the user is best qualified to judge the magnitude of tolerable errors. For estimating a flood-crest elevation, precision might be quite unnecessary; in designing a submerged sill to regulate the Great Lakes, accuracy of the highest order might be essential. It is believed that the experimenter is under obligation to present data with meticulous precision, if possible in a form which enables the designer to make such short cuts as the specific application may justify. A statement to the effect that most uses do not justify great precision is true but hardly necessary—this fact is well known and universally accepted. Objection to an accurate analysis which permits the designer to make an application consistent with his purpose, and without disadvantage, is not considered indicative of a "practical standpoint."

A careful distinction should be made between a legitimate simplification and a rough approximation adopted only to avoid computation. A simplification is proper when the materials dealt with have physical properties which are so complex or variable that an involved equation is unwarranted for physical reasons and undesirable because it obscures significant relationships. Mr. Blaisdell's suggestion is not based on physical reasons; nor does it result in a

simplification warranted by the loss of accuracy involved. Its proposal can be based only upon an unwarranted mistrust of the reliability of hydraulic models or of the accuracy obtainable in the design of hydraulic structures.

A number of the discussers have commented favorably upon various indications of accuracy in the experimental work. The writer wishes to emphasize, as stated in "Acknowledgment," that the model tests were conducted by Mr. Morgan.

There were no comments on the writer's statement that the discrepancy between model and prototype is probably chargeable to the latter. Five years have passed since, in 1938, this statement was made in the original draft of the paper. The writer's present belief is that the discharge indicated by a well-designed spillway model, when the model head exceeds 3 in. or 4 in., is fully as accurate as that obtainable by any field method in current use.

The data presented in the discussions fully confirm the conclusion that the effect of submergence upon the discharge is a function of spillway shape. This fact has not always been as evident as it now appears; once the writer was advised to check some submerged spillway data very carefully because they did not agree with the U. S. Deep Waterways curve. This suggestion was influenced by the agreement between Mr. Walton's data and the U. S. Deep Waterways tests. Although Mr. Walton's spillway and the U. S. Deep Waterway's spillway differ in shape, they have a common characteristic. This is the factor believed to control the effect of submergence upon the discharge—namely, the distribution of pressure within the nappe at or near the spillway crest. Since the nappe springs free of these spillways or tends to do so, the occurrence of subatmospheric pressure beneath and within the nappe affects the head-discharge relationship. Submergence of these spillways tends to modify pressure conditions radically at the crest, making them "submergence sensitive." When the nappe is in contact with the spillway at all points with atmospheric pressure at the water-masonry interface, or particularly when the nappe exerts a pressure against the spillway, submergence has a smaller effect upon the pressure distribution at the crest. For this reason, a dam with a stable nappe is less affected by submergence. This is the case on the spillways tested by Professor Cox, Professor Mavis, Messrs. Nelson and Hartigan, and the writer. The difference in the submergence effect in the two crests tested by Messrs. Nelson and Hartigan confirms this idea. A comparison of the several spillway sections reported indicates that this explanation is reasonable. (It should be observed that the vertical face of the U. S. Deep Waterways section is on its downstream side.)

If (as the writer believes) the data brought to light in the discussions is indicative of the wealth of information available from the personal files of engineers, the mere announcement, for discussion, of subjects carefully chosen by the appropriate committees may be a more effective method of developing references upon suitable problems than waiting—sometimes many years—for an author to introduce a subject concerning which information is needed badly. In the present case, the prevailing method of computing the effect of submergence upon the discharge actually produced errors, when applied to modern spillways, greater than the effect of completely ignoring the submergence.

That this observation can now be made as an established fact rather than as an isolated opinion is due to the efforts of those who presented discussions. The writer is pleased in having had a small part in correcting this situation and in making more reliable quantitative data generally available.

FREDERICK R. BROWN,[43] JUN. AM. SOC. C. E.[43a]—Most of the papers and discussions on model-prototype conformity have dealt with hydraulic structures and tests on models constructed geometrically similar to their prototype counterparts. There are other types of model studies, however, which were given little emphasis in the Symposium. This is probably due to the fact that few engineers have tried to study the model-prototype conformity that exists for geometrically distorted models constructed for the solution of problems on channel capacity and bed movement.

Models of hydraulic structures practically always are constructed to the same horizontal and vertical scales, and dynamic similitude is attained in so far as possible by operating these models in accordance with the Froudian scale relationships. In most cases a large model scale must be used to satisfy the roughness ratio requirement which can be shown to be equal to $(L_r)^{1/6}$; thus, as the model scale decreases, the smoothness of the model must be increased. Geometric similitude must be maintained because it is important to the success of the study that the magnitude and direction of prototype forces and flow paths be reproduced accurately.

In the case of some river models, however, where the area to be reproduced is large and it is necessary that the vertical scale be larger than the horizontal scale to obtain measurable quantities on the model or where it is necessary to add an additional slope distortion for movement of bed material, neither geometric similitude nor correct dynamic similitude is obtained. Complete similitude is not essential if care is taken in selecting the model scales for solution of the problem at hand. If the problem is one involving channel capacities or study of flood-crest profiles, the model scales can be distorted considerably inasmuch as there is no need for the exact reproduction of velocity distribution, paths of flow, etc. All that is necessary to insure reliable model results in the foregoing case is to adjust the model roughness in order that a discharge and velocity scale will be obtained that will permit the reproduction of the desired stage-discharge relations and the average change in potential and kinetic energy at varying cross sections. If the problem involves the movement of bed material, the distortion of model scales must not be too large. The success of this type of model depends upon the reproduction of the proper velocity distribution for movement of the bed material. If the horizontal and vertical scales selected are such that the distortion is low and the stream is wide with respect to the depth, the distortion will not alter the general shape of the channel, and close similarity of velocity distribution will exist.

In the case of the Mississippi River channel model described in the Symposium, the horizontal scale was 1 : 600 and the vertical scale was 1 : 150

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[43a] Received by the Secretary August 2, 1943.

which gives a distortion of 4. This distortion was small enough so that the velocity distribution and current directions were reproduced accurately. No tests of improvement plans were undertaken, however, until the accuracy of the model was insured. The accuracy of the model was established by a verification procedure in which model operating conditions were developed such that the movement of bed material in the model was consistent with that in the prototype. In order to provide adequate movement of the coal bed material, it was found necessary to provide a slight additional slope distortion. Instead of altering the slope, either the discharge scales or the time scales or both could have been adjusted to reproduce the prototype bed movement—that is, the discharge scale or the length of run at low stages could be increased over that used for high stages.

Mr. Einstein states that, during a visit to the U. S. Waterways Experiment Station at Vicksburg, he was astonished to see how much the natural roughness of a model must be increased to counteract the effect of the distortion on friction. This raised the question in his mind of the reliability of distorted hydraulic models.

The actual roughness used in distorted channel models at the Experiment Station is placed in accordance with accepted model practice and can be shown to be mathematically correct. If the linear scale and the distortion is small, however, it may become impossible to make the model smooth enough. On the other hand, if the distortion is too large, the model cannot be made rough enough. Between the two aforementioned extremes, there are model scales that will permit the use of the theoretical roughness ratio $[n_r = (R_r)^{2/3} (L_r)^{-1/2}]$, in which R_r is ratio of the hydraulic radii for the model (R_m) to the prototype

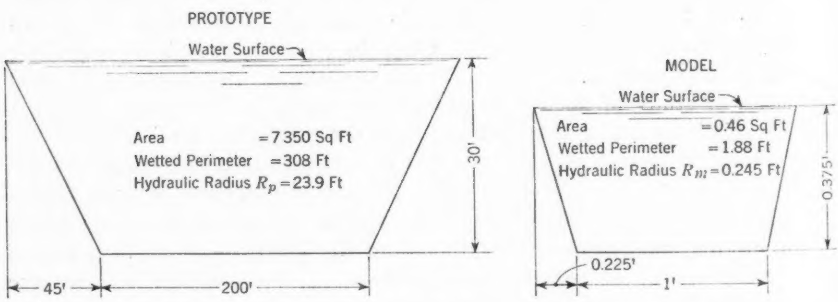

FIG. 87

(R_p). It is not necessary, however, to use the theoretical roughness values inasmuch as a number of discharge scales with their corresponding roughness values will provide identical flow lines.

An excellent example to demonstrate that it is possible to have various combinations of roughness and discharge scales for the reproduction of identical flow lines was computed by Lieutenant Tiffany (78). It was shown that,

with a model constructed to a horizontal scale ratio of 1 : 200 and a vertical scale ratio of 1 : 80, the following relationships would obtain for the channel whose cross section is shown in Fig. 87: $R_r = \frac{R_m}{R_p} = \frac{0.245}{23.9} = \frac{1}{97.5}$; and

$$Q_r = (L\,y\,V)_r = \left(\frac{R^{2/3}\,L^{1/2}\,y^{3/2}}{n}\right)_r = \frac{1}{n_r} \times \left(\frac{1}{97.5}\right)^{2/3} \times \left(\frac{1}{200}\right)^{1/2} \times \left(\frac{1}{80}\right)^{3/2} = \frac{1}{n_r} \times \frac{1}{215{,}000}.$$

Thus, from the foregoing computations it can be seen that the discharge and roughness ratios are dependent upon each other. Assuming the prototype roughness to be about 0.024, the following conditions would obtain:

Q_r	n_r	n_m
1 : 120,000	1 : 1.79	0.0134
1 : 128,000	1 : 1.68	0.0143
1 : 143,000	1 : 1.50	0.0160
1 : 150,000	1 : 1.43	0.0168

In this case the theoretical discharge scale of 1 : 143,000 $[Q_r = L_r\,(y_r)^{3/2}]$ could have been used in the model since a model roughness of 0.0160 is possible of attainment. However, in the event that the model roughness indicated by the theoretical relationship was too rough, the model could have been roughened as much as practical and the discharge scale changed accordingly to reproduce the prototype flow lines.

For further detailed information regarding the accuracy of distorted models and the verification procedure, for movable bed models, reference is made to the Society's Manual on "Hydraulic Models" (78*a*). It is recommended that any one connected with model investigations familiarize himself with this excellent dissertation on both distorted and undistorted models.

All in all, the distorted model provides a valuable tool for the solution of difficult problems. If the tool is handled correctly, a quantitative interpretation of the results pertaining to the problem for which the model was designed is possible. The departure from conditions of similitude, however, makes it necessary to qualify other results or information. Experienced model technicians realize the true value of distorted models and can be depended upon to interpret the model results correctly.

Comparable model and prototype data are difficult to obtain for channel models inasmuch as the structures or plans developed through model investigation often are not installed or have not had time since construction to indicate changes caused by their installation. Of the amount of information available, however, the prototype performance is bearing out the predictions of the distorted scale models, in practically every case.

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(50) **"Submerged Weir Formula Verified,"** by G. N. Cox, *Engineering News-Record*, Vol. 123, No. 9, August 31, 1939, p. 52.

(77) **"Studies of Crests for Overfall Dams,"** *Bulletin No. 3*, Pt. VI, Hydraulic Investigations, Boulder Canyon Final Repts., U. S. Dept. of the Interior, Bureau of Reclamation, Denver, Colo., December 31, 1942 (unpublished).

(78) **"Hydraulic Models,"** *Manual of Engineering Practice No. 25*, Am. Soc. C. E., 1942, p. 39, Fig. 2. (*a*) Section V, p. 36.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

FLOW CHARACTERISTICS AT RECTANGULAR OPEN-CHANNEL JUNCTIONS

Discussion

BY MESSRS. I. B. HOSIG, L. STANDISH HALL, AND BROOKS T. MORRIS

I. B. HOSIG,[12] ESQ.[12a]—To the writer this paper is of interest principally because it presents an additional case in which the application of the principles of analysis by forces yields results in an hydrodynamic problem. It is gratifying to find this method of analysis being used by the young men entering hydraulic work and, although, to date, its practical applications have been few, it has elucidated with brilliancy the problem of the hydraulic jump—for instance, where work with the principles of analysis by energy content had enveloped that problem in a fog of empiricism.

The author modestly disclaims a solution of great practical value because the theory is adequately bolstered only by experiments in which the angle between the two intersecting streams is 45°. However, it is probable that in cases in nature the angle will be found to be that or less, and thus the work is practical.

In the irrigation and drainage work with which the writer is familiar, the case of division of water is entirely ruled out of the class of problem herein discussed because division is under control by means of structures. Division is a major work in the control of irrigation water.

The joining of streams in irrigation work consists principally of taking storm water or return flow into supply canals. The water surface of the principal stream is usually much lower than that of the incoming stream and the volume is also much greater. Hence the question of the elevation of the joined waters is of no interest; but the problem of the force of the incoming water is of paramount importance.

In the case of drainage channels the resultant water surface at the junction may be of considerable importance. In the writer's practice the method of

NOTE.—This paper by Edward H. Taylor, Jun. Am. Soc. C. E., was published in November, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1943, by Harold K. Palmer; and September, 1943, by Messrs. G. H. Hickox, J. C. Stevens, and C. J. Posey.

[12] Engr., Bureau of Reclamation, Denver, Colo.

[12a] Received by the Secretary July 26, 1943.

treating the problem has been to make the channel below the junction of such size that it will handle the anticipated flow with the water surface at the junction at the same elevation as that of the incoming channels, assuming flow in each branch to follow Chézy's formula where the anticipated flows are of consequence. Otherwise, the bottom grades are all brought to one elevation. In the writer's practice observation of the behavior of drains at junctions has not been very illuminating as discharges are difficult to predict and are highly variable; hence flows observed are usually not those assumed when making the layout but the observations are clear in one respect—that the forces involved govern resultant channel conditions. It is sometimes necessary either to direct the combined channel as a resultant of the two branches or to contemplate additional work in the nature of bank protection which is in effect the introduction of additional force. This leads to the idea that more fruitful results will be obtained if the problem is presented in such a way as to recognize all forces of such magnitude as can appreciably affect the motions of the body involved.

In the case in the paper where two streams meet at an angle of 45°, the transverse (with respect to the straight channel) component of the $\frac{Q\,v}{g}$ force of the stream coming in at an angle was not included in the mathematical treatment. The author recognized the omission but the photographs show that the force does have an effect on flow conditions. The writer believes that it is this force which limits the use of the mathematical treatment presented to angles of an order of 45° or less for the case considered. This is not a criticism which is intended to imply that analysis by forces in this case is a weak approach to the problem. In fact it appears to be the only analytical approach and the proper course is a more comprehensive setup. Model study probably would solve a particular problem, but model study without analysis is a barren plant in the field of engineering science.

L. Standish Hall,[13] M. Am. Soc. C. E.[13a]—Little research has been done on the phase of hydraulics described in this paper. Many variables are involved in the solution of the problem, one of which has been eliminated in the paper by making the bed horizontal. The flat grade produces low velocities and, consequently, small values of kineticity.

In many of the design problems involving converging channels, the writer has found that the velocities have been great enough to require paving or a lined channel. The velocities often are greater than the critical and the kineticity greater than 2. With kineticity of this magnitude, the depth of flow above the junction is not affected by backwater from the combined flow; hence, the depth of flow in the two converging channels is not necessarily the same. When the depths are different, cross waves will develop that are transmitted downstream, sometimes with undesirable results.

Even with the velocities below critical it would be difficult to project the curves to include higher kineticity on the basis of the data presented in the paper.

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[13a] Received by the Secretary July 29, 1943.

For the data plotted in Fig. 3, it should be noted that, for small values of n_q, n_y would be independent of the value of k_2. For this reason, the intersection characteristics should be plotted against k_3.

If the flow from the side channel becomes zero, the ratio $n_q = 0$. Then, from Eq. 5,

$$(n_y - 1)[(n_y)^2 + n_y - 4\,k_3] = 0 \qquad (8)$$

With $n_q = 0$, assuming that wall friction can be neglected, the depth ratio, $n_y = \frac{y_a}{y_b} = 1$ can only occur when $k_3 = \frac{1}{2}$; or, expressed in words, under the condition of no flow in the side channel, the velocity head is always one half of the depth. This may be true, but it is difficult to believe. If Eq. 8 is solved for n_y, then

$$n_y = \frac{-1 \pm \sqrt{1 + 16\,k_3}}{2} \qquad (9)$$

From Eq. 9, it is seen that one root is positive and one negative; and with kineticity less than $\frac{1}{2}$, such as was observed in the author's experiments, the value of n_y should be less than unity. In other words, a jump should occur at very low velocities. This is contrary to accepted hydraulic theory, as a jump can only occur when k_3 is greater than $\frac{1}{2}$.[14] The solution to this dilemma lies in the fact that in Eq. 5, when $n_y = 1$ and $n_q = 0$, the term containing k_3 drops from the equation. Hence, for this special case the kineticity is immaterial, and Eq. 9 can apply only when $n_y \neq 1$.

For the other extreme, if all of the flow originates from the side channel, $n_q = 1$. Then, from Eq. 5, if $\theta = 90°$, the relation

$$n_y = \sqrt{1 + 4\,k_3} \qquad (10a)$$

is obtained and, from Eq. 6,

$$k_2 = \frac{k_3}{(1 + 4\,k_3)^{1.5}} \qquad (10b)$$

These relations indicate that depths in channels 1 and 2 (Fig. 2) would be greater than in channel 3, and k_2 would be less than k_3. The lack of agreement between theory and experiment shown in Fig. 3 makes the application of Eqs. 10 questionable. The results of experiments covering this condition would be interesting if available.

For converging channels, the lack of agreement of the results with the momentum equation when $\theta = 135°$ would suggest that a comparison of the energy losses might yield a more satisfactory solution. In Fig. 3, when $n_q = 0.4$, the relation between $\frac{y_a}{y_b}\left(\text{or } \frac{y_1}{y_2}\right)$ and k_2 is identical for both $\theta = 45°$ and $\theta = 135°$. With $n_q = 0.6$ there is very little difference in the relation with either angle. Only when $n_q = 0.8$—that is, with 80% of the flow entering from the side channel—is definite difference in the relation apparent between supplementary angles of convergence. The results would indicate that the

[14] "The Hydraulic Jump in Terms of Dynamic Similarity," by Boris A. Bakhmeteff and Arthur E. Matzke, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 630.

momentum of translation is converted into a momentum of rotation, and, hence, with small flows in the side channel, the angle of entrance is immaterial.

An example of dividing flow is given by a side spillway for regulating the diversion into a canal system. The conditions are not parallel with those given by the author since the bottom of the side channel is at a higher elevation than the bottom of the main canal. The momentum principle can be used in solving this problem. In the case of the change of momentum affecting the water surface profile in such a canal, it has been assumed that momentum is lost with the overflowing water.[15]

On the other hand, in a side-channel spillway from a reservoir (an example of combining flow), it has been assumed that the momentum of the water discharging over the spillway lip is entirely lost upon impact with the water in the spillway channel. This assumption has been made since the flow moves over the crest of the weir in a direction approximately at right angles to the axis of the spillway channel.[16] This would correspond with the assumption made by the author in Eq. 2.

The variables affecting combining and dividing flow are so numerous that no general relation applicable to all conditions can be found. In most cases, when a prior knowledge of the flow characteristics at an intersection is important, a model should be constructed as suggested by the author. General relations applicable to specific cases may be obtained where it is possible to control a sufficient number of the variables. The results presented in this paper are a step in the right direction.

Brooks T. Morris,[17] Jun. Am. Soc. C. E.[17a]—Experiments on so general a problem as channel junction performance cannot be expected to cover more than a fraction of the field. Within the limits of his apparatus and of the time given to its operation, Mr. Taylor has developed good data. The quantities employed in both combining and dividing streams cover the range of tranquil flow nearly to the limit of critical velocity in the most heavily loaded conduit.

The analyses attempted by Mr. Taylor make the data of considerable use in revealing the weaknesses of certain assumptions which are customarily employed in hydraulic computations but which are inappropriate to problems of combining and dividing flow. The first of these is the assumption of uniform velocity distribution over the channel cross sections. Theoretical analysis and long experience each have shown that this assumption is adequate in ordinary pipe and channel calculations where similar, although not strictly uniform, velocity distributions exist at the various sections for which force-balance equations are written. On the other hand, within individual cross sections, manifold pipes and branched channels contain extreme variations of velocity distribution. Such conditions, like the 135° intersection, contain stream filaments moving in opposite directions in the same cross section. To describe such conditions, formulas like Eq. 2 should contain distribution coefficients corresponding to the

[15] "Side Spillways for Regulating Diversion Canals," by W. H. R. Nimmo, *Transactions*, Am. Soc. C. E., Vol. 92 (1928), p. 1561.

[16] "Side Channel Spillways," by Julian Hinds, *ibid.*, Vol. 89 (1926), p. 881.

[17] Research Engr., C. F. Braun & Co., Alhambra, Calif.

[17a] Received by the Secretary August 16, 1943.

ratio of the integrated mean momentum to that calculated from the mean velocity of flow. As long as those coefficients approach unity, as in the 45° junction, the analysis as presented by the author is successful.

The second familiar assumption is the dual one of hydrostatic pressure distribution and of horizontal surface at each cross section. This assumption leaves the author short one equation in studying the dividing flow problem and makes necessary his assumption (3) regarding the depths in combining flow. Except for this, the fact of a gradient of depth, or pressure, gradient across the 1–3 channel might be recognized and the second equation of momentum might be written. The writer recognizes the difficulties inherent in writing this equation, but he believes that it is a necessary step in the rational description of combining or dividing flow. Also, the determination of momentum distribution coefficients suited to varying junction geometry is not easy. The continuing studies on the mixing of parallel streams should be informative in this regard. Excellent investigations of certain types of such mixing have been made already by W. Tollmien and A. M. Kuethe.[18]

The writer felt handicapped by the coefficient k until he recognized it as merely another form of the Froude number or kineticity of flow such that

$$k = \frac{V^2}{2\,g\,y} = \frac{\mathbf{F}^2}{2} = \frac{\lambda}{2} = \frac{1}{2}\left(\frac{y_c}{y}\right)^3 \qquad (11)$$

Mr. Bickerstaff's photographs are very helpful in picturing the conditions in the author's experiments. However, the writer is concerned over the effects that departure of the dye interface from the vertical might have on the interpretation of the illustrations. Was the dyed stream only a shallow layer or did it occupy the full depth of the section? If the latter was the case, did the interface remain vertical?

The verity of Mr. Taylor's data is undeniable, and his conclusions have been drawn with care. Still, the writer would be sorry to see either used to discourage further attempts at theoretical analysis of the problem. It is for that reason that he has sought to show the weaknesses of the underlying assumptions.

Corrections for *Transactions:* In November, 1942, *Proceedings*, page 1524, ordinate caption for Fig. 3, change "$\frac{y_1}{y_2}$" to "n_y"; in Figs. 4(a) and 4(b), interchange the numerals "2" and "3" to agree with Fig. 2.

[18] "Modern Developments in Fluid Dynamics," by S. Goldstein, Oxford, 1938, p. 592.

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DISCUSSIONS

EFFECT OF TURBULENCE ON SEDIMENTATION

Discussion

By Messrs. Garbis H. Keulegan, C. W. Thornthwaite, and A. A. Kalinske

Garbis H. Keulegan,[8] Esq.[8a]—The statement of the boundary conditions relating to the solution of the differential equation of the one-dimensional case requires examination.

Apparently it is the author's understanding that the solution of the basic equation, Eq. 17*c*, requires four boundary conditions. The equilibrium distribution which finally is established is taken as one of these. Obviously, the final disposition of a physical process is a consequence and cannot be preassigned as a boundary condition. Indeed the description of the final disposition must result from the analysis itself. Actually, only three conditions are necessary for the desired solution.

The mathematical expression of the argument is much simplified if Eq. 17*c* is transformed into a relation involving dimensionless quantities only. Suppose that t is measured in terms of T, and y in terms of h; thus, let $\tau = \frac{t}{T}$; $\eta = \frac{y}{h}$; $T = \frac{h^2}{\epsilon}$, and $\beta = \frac{w}{h\,\epsilon}$. Then, Eq. 17*c* takes the form

$$\frac{\partial c}{\partial \tau} = \frac{\partial^2 c}{\partial \eta^2} + \beta \frac{\partial c}{\partial \eta} \qquad (54)$$

Suppose that a solution is sought, subject to the following three boundary conditions:

First, the transport of the material across the upper surface into the liquid below is brought about in a preassigned manner. Mathematically,

$$\frac{\partial c}{\partial \eta} + \beta\, c = \phi(\tau); \qquad \eta = 1, \quad \tau > 0 \qquad (55)$$

Note.—This paper by William E. Dobbins, Jun. Am. Soc. C. E., was published in February, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1943, by A. M. Gaudin; May, 1943, by John S. McNown; and June, 1943, by Messrs. H. A. Einstein, and Thomas R. Camp.

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[8a] Received by the Secretary June 17, 1943.

Second, the rate of pickup from the bottom is controlled in a preassigned manner; or, what is the same thing, the gradient of concentration at the bottom is controlled in a preassigned manner. Mathematically,

$$\frac{\partial c}{\partial \eta} = \theta(\tau); \qquad \eta = 0, \quad \tau > 0 \ldots\ldots\ldots\ldots\ldots\ldots (56)$$

Third, the initial concentration is specified. Mathematically,

$$c = f(\eta); \qquad 0 > \eta > 1, \quad \tau = 0 \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

It will be noted that the conditions assumed are of the most general type.

If a solution of Eq. 54 conforms to the end conditions, Eqs. 55, 56, and 57, then this solution is unique.

In order to prove this statement, a theorem of expansion is needed. Consider the sequence of functions

$$\Theta_n = 2\,\delta_n \cos \delta_n\,\eta + \beta \sin \delta_n\,\eta; \qquad n = 1, 2, \cdots \quad \ldots\ldots\ldots (58)$$

in which δ_n is one of the positive roots of the relation

$$4\,\delta\,\beta \cos \delta = (4\,\delta^2 - \beta^2) \sin \delta \ldots\ldots\ldots\ldots\ldots\ldots (59)$$

These functions have the properties

$$\int_0^1 (\Theta_n)^2\, d\eta = \frac{(2\,\delta_n + \beta)^2}{2} \ldots\ldots\ldots\ldots\ldots\ldots (60a)$$

and

$$\int_0^1 \Theta_n \Theta_m\, d\eta = 0; \qquad n \neq m \ldots\ldots\ldots\ldots\ldots (60b)$$

Because of these properties (ordinarily referred to as the properties of orthogonality) any bounded function $F(\eta)$ may be expanded in terms of Θ_n in the interval, $0 \leqq \eta \leqq 1$, provided that, if the function has discontinuities, these are finite in number. Mathematically,

$$F(\eta) = \sum (C_n \Theta_n) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

in which $C_n = \dfrac{2}{(2\,\delta_n + \beta)^2} \displaystyle\int_0^1 F(\eta)\,\Theta_n\, d\eta$. In addition to the discontinuities, the expansion may fail at the end points, $\eta = 0$ and $\eta = 1$, depending on the character of F. Similar and equivalent expansions were used by the author.

Now suppose that the solution of Eq. 54, conforming to the boundary conditions, Eqs. 55, 56, and 57, is not unique. Let C_1 be one solution and C_2 another. Consider the difference,

$$q = C_1 - C_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (62)$$

Since C_1 and C_2 individually satisfy the boundary conditions, Eqs. 55, 56, and 57, it can be shown that q satisfies certain simpler boundary conditions, which

are derived from Eqs. 55, 56, and 57:

$$\frac{\partial q}{\partial \eta} + \beta\, q = 0; \qquad \eta = 1, \quad \tau > 0 \ldots\ldots\ldots\ldots\ldots (63a)$$

$$\frac{\partial q}{\partial \eta} = 0; \qquad \eta = 0, \quad \tau > 0 \ldots\ldots\ldots\ldots\ldots (63b)$$

and

$$q = 0; \qquad 0 \leqq \eta \leqq 1, \quad \tau = 0 \ldots\ldots\ldots\ldots\ldots (63c)$$

Again, since C_1 and C_2 individually satisfy the linear differential equation, Eq. 54, q also satisfies the same equation,

$$\frac{\partial q}{\partial \tau} = \frac{\partial^2 q}{\partial \eta^2} + \beta\, \frac{\partial q}{\partial \eta} \ldots\ldots\ldots\ldots\ldots (64)$$

Obviously, the solution of Eq. 64 conforming to the boundary conditions, Eqs. 63*a* and 63*b*, is

$$q = \sum (C_n\, e^{\gamma \tau}\, e^{-0.5 \beta^2 \eta}\, \Theta_n) \ldots\ldots\ldots\ldots\ldots (65)$$

in which $\gamma = -\frac{\beta^2}{4} - \delta^2$. It remains to ascertain C so that the boundary condition given by Eq. 63*c* is also satisfied. When $\tau = 0$,

$$q = \sum (C_n\, e^{-0.5 \beta^2 \eta}\, \Theta_n) \ldots\ldots\ldots\ldots\ldots (66)$$

Since $q = 0$ at the points $\eta = 0$ and $\eta = 1$, an application of the theorem underlying Eq. 61 yields

$$C_n = 0; \qquad n = 1, 2, \cdots \quad \ldots\ldots\ldots\ldots\ldots (67a)$$

and, therefore,

$$q = 0; \qquad \tau > 0 \ldots\ldots\ldots\ldots\ldots (67b)$$

Accordingly $C_1 = C_2$. This proves the proposition that the solution of Eq. 54 conforming to the boundary conditions, Eqs. 55, 56, and 57, is unique.

Thus, the fourth boundary condition stated by the author is redundant for the problem since the solution finally arrived at is correct. That the solution is correct may be established by modifying the arguments of the author so that only the first three conditions are used to obtain the final solution; or another method of solution may be adopted, using the same three conditions and leading to the same results. Both of these methods were followed by the writer.

C. W. THORNTHWAITE,[9] ESQ.[9a]—In connection with studies directed toward the objective of measuring the transport of moisture from a natural surface into the atmosphere, through the medium of turbulent mass exchange, the writer and his associates have found it necessary to make detailed and careful observations of the variation of wind velocity with height in the layer within 30 ft of the ground (20)(21)(22).[9b] To obtain observations that would reveal

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[9a] Received by the Secretary June 24, 1943.

[9b] Numerals in parentheses, thus: (20), refer to corresponding items in the Bibliography (see Appendix 1) of the paper, and at the end of discussion in this issue.

the true nature of the wind gradient it was first necessary to select anemometers that were identical as to starting and stopping speeds, and running speed at all velocities. A number of matched anemometers were mounted one above the other on a 28-ft tower and connected electrically to a photographic recorder that makes simultaneous recordings of all at 15-min intervals.

The data indicate that the familiar logarithmic law of wind velocity is of restricted validity. This law applies very well when the temperature lapse rate is adiabatic (or within this thin layer, isothermal), but it fails when the lapse rate is superadiabatic and also at times of inversions. Gradients of wind movement for a series of hours on July 25 and 26, 1942 (Fig. 18), illustrate the dependence of wind distribution on stability. The data are plotted

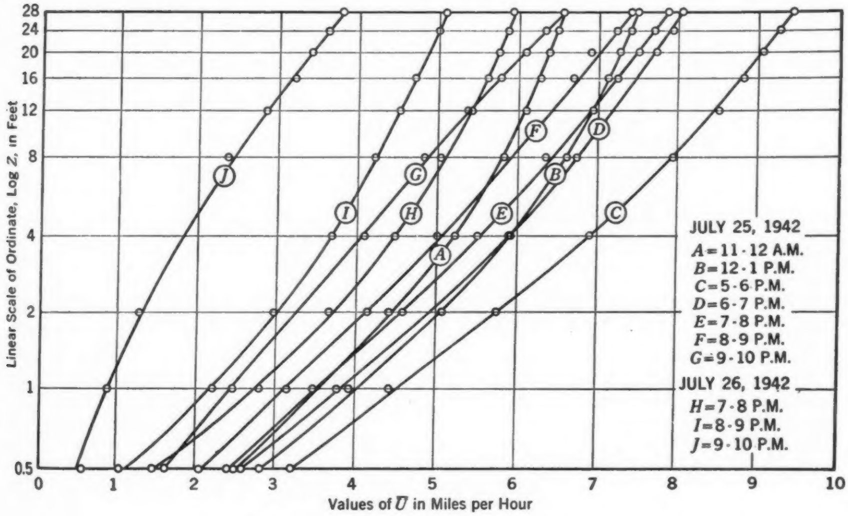

FIG. 18.—GRADIENTS OF WIND MOVEMENT

on a semilogarithmic scale; in order for the logarithmic law to be verified the data for individual hours would form straight lines; but only two series made in the evening of July 25 (lines F and G) approach straight lines. The daytime observations plot along lines having concave curvature; those made near noon show the greatest departure from a straight line. Observation J made near midnight displays strong convex curvature. This is the only one of the series in the figure that does so, but it represents a tendency that is characteristic of most observations made in the early hours of the morning before sunrise.

These observations suggest a reasonable physical picture of turbulent mixing in the atmosphere. When the thermal lapse rate is adiabatic, as for brief periods in the morning and again in the evening, and the logarithmic law is fulfilled, Prandtl's theory (Eq. 1) is valid, and the coefficient of turbulent mixing, designated ϵ by the author, may be obtained by Eq. 9. Turbulent mixing is mechanical only.

In the middle of the day, when thermal convection is active, convective turbulence is superimposed upon mechanical turbulence and ϵ becomes greater —possibly much greater than is indicated by Eq. 9. The concave curvature of the wind gradients is a consequence of this increase in the coefficient of turbulent mixing. As momentum is transported downward more actively the tendency for differences in velocity in adjacent layers to be equalized increases, and the wind profile becomes more nearly vertical except very close to the ground surface.

At night when the ground, having been cooled by radiation, cools the adjacent air, the lowest layer of air becomes cooler than that above it and a stable lapse rate is created. The air next to the ground, being cooler, is denser than that above and, if moved mechanically upward into a faster moving layer, would tend to move down again instead of mixing with the air in the new environment. In this instance ϵ becomes less than is indicated by Eq. 9. Momentum is transported downward less actively, the tendency for velocity differences in adjacent layers to be equalized diminishes, and the velocity profile departs more than ever from the vertical except within a foot or two of the ground.

This physical picture of the variation of ϵ with stability is in accord with the well-known phenomenon that the diurnal march of wind velocity a few hundred feet aloft (on the top of the Eiffel Tower, for example) is opposite to that close to the ground.

The suspension of sediment in water will tend to make the fluid more stable. The experiments by Mr. Vanoni (7) confirm this. Mr. Vanoni's semilogarithmic plots of velocity profiles at the center of a flume depart quite widely from the logarithmic law and display the same convex curvature that has been observed in a stable atmosphere. This is contrary to his statement that, "The data follow the logarithmic law very closely except near the bottom where the measured velocities exceed those predicted from the law" (7*a*).

Mr. Vanoni reports that the velocity of sediment laden water is greater than that of clear water, which fact he correctly ascribes to a decrease in ϵ. He thinks that the decrease in ϵ is due to a decrease in the value of K, the von Kármán universal turbulence constant. Of course, ϵ will decrease with increasing density (see Eq. 9) but at least in the free air density differences in the profile of stable air are so small that their effect on ϵ must be negligible. The increase of density due to sediment suspension likewise falls far short of being sufficient to account for the decrease in ϵ.

The data plotted in Fig. 7 suggest a variation in ϵ from top to bottom of the tank that may be due to a changing value of the mixing length with changing sediment concentration. Turbulence was imposed artificially by a "lattice structure" and the author assumes that the fluid was mixed uniformly. Actually the sediment concentration near the bottom of the cylinder was about three times as great as at the top at the beginning of the experiment. Therefore, in line with Mr. Vanoni's findings, the value of ϵ near the bottom would be less and that near the top would be greater than the mean value. Then, when the apparatus is set in motion, mixing near the bottom is less than expected and the sediment concentration does not diminish at the rate called

for by Eq. 41. Conversely, near the top of the cylinder, mixing is greater and the concentration of sediment is in excess of that predicted. Only when the sediment is nearly uniformly distributed through the fluid do the author's data verify his theory.

An interesting experiment would be to simulate, in a flume, the unstable conditions that prevail in the atmosphere near the ground on warm sunny days. Heating coils in the bottom of the flume would serve the purpose, and it is to be expected that plots of the velocity gradient on semilogarithmic paper would display the same concave curvature that has been observed in the wind velocity profiles.

The writer is of the opinion that Prandtl's theory of turbulence will need to be modified before it can be applied to the study of sediment transport in streams, since there seems to be considerable evidence that the mixing length varies with height in a sediment laden fluid, according to a law that is different from the one that has been applied successfully to a fluid that is homogeneous throughout.

A. A. KALINSKE,[10] ASSOC. M. AM. SOC. C. E.[10a]—By analyzing an especially difficult problem mathematically and then testing the validity of his development by making experiments under certain controlled boundary conditions, the author has presented a very fine contribution. His analyses and experiments lead him to certain definite conclusions. These all seem to be warranted by the facts at hand and the writer is in entire agreement with them. Of particular significance to the writer are the author's conclusions relating to the influence of the bed composition on the suspended material. This item is of basic importance in developing a relation among the absolute value of suspended material concentration, the bed composition, and the hydraulic conditions at the bed.

In connection with the author's assumption of similarity in turbulent flow between the momentum and sediment transfer processes ($\beta = 1$) the results of further studies that have been made along this line may be of interest (23). A detailed study was made of the value of the mass-transfer coefficient in turbulent flow in open channels, and its directly measured value was used to calculate sediment distribution curves. The calculated sediment distribution curves agreed very well with actual sediment concentration measurements. The turbulence diffusion coefficients were measured by determining the spread in the turbulent flow of an injected mixture of hydrochloric acid and alcohol, the proportions being adjusted so the density of the mixture was equal to that of the water. Point samples of water were taken to determine the spread of the mixture, and the concentration of the chlorides in the samples was determined by titration. This provided a quick and accurate means of obtaining data for direct measurement of the turbulence diffusion coefficient.

The author's comments on the "pickup" of material from the bed and the relation between suspended and bed material were of considerable interest to the writer. This is one of the most important problems relating to suspended

[10] Associate Director, Iowa Inst. for Hydr. Research, Iowa City, Iowa.

[10a] Received by the Secretary July 19, 1943.

sediment transportation. There is no doubt but that, for equilibrium conditions, there exists, for any material size, a definite relation between the amount of that material on the bed, the amount in suspension above the bed, and the hydraulic conditions at the bed. The first attempt to formalize this relation was made by E. W. Lane, M. Am. Soc. C. E., and the writer (12), and certain field data were presented indicating the existence of such a relation. Since 1939, laboratory experiments have been made which have verified this further and have indicated more definitely the parameters that enter into the relation. Experiments with very fine materials also have indicated the existence of such a relation (24). The general functional formula is as follows:

$$\frac{c_0}{c_b} = \phi\left(\frac{w}{\sqrt{\frac{\tau_0}{\rho}}}, \frac{D\sqrt{\frac{\tau_0}{\rho}}}{\nu}\right) \qquad (68)$$

in which c_0 and c_b are the concentrations in weight per unit volume of the sediment and liquid mixture for sediment characterized by fall velocity, w, and size, D, in suspension and in the bed, respectively. Laboratory data for materials ranging in size from 0.001 mm to 0.30 mm have been obtained and these data follow the functional relationship indicated by Eq. 68.

Another item that seems to merit some consideration is the possibility of obtaining solutions to complex differential equations, such as enter into the problem considered by the author, by methods of numerical integration. The integration of Eq. 17*c*, even under simplified boundary conditions and assuming a constant value for ϵ, is a most formidable mathematical problem. Also, the final solution is so complex as to discourage its use. Methods of numerical integration have been used with success in many physical problems such as, for instance, unsteady heat flow; and, although sometimes the calculations are laborious, they are always simple and can be done by nontechnical workers.

To illustrate how numerical integration may be applied to this problem the solution to Eq. 17*c* for the two boundary conditions will be indicated. Consider a point y distance above the bottom and two points which are (Δy) above and below and which will be designated a and b, respectively. Then the values of $\frac{\partial c}{\partial y}$, $\frac{\partial^2 c}{\partial y^2}$, and $\frac{\partial c}{\partial t}$ at the point y can be written in increment form as follows:

$$\frac{\Delta c}{\Delta y} = \frac{c_a(t) - c_y(t)}{\Delta y} \qquad (69a)$$

$$\frac{\Delta^2 c}{\Delta y^2} = \frac{c_a(t) - 2\,c_y(t) + c_b(t)}{(\Delta y)^2} \qquad (69b)$$

and

$$\frac{\Delta c}{\Delta t} = \frac{c_y(t + \Delta t) - c_y(t)}{\Delta t} \qquad (69c)$$

Substituting Eq. 69 into Eq. 17*c* and letting $\Delta t = \frac{(\Delta y)^2}{2\,\epsilon}$:

$$(c_y)' = \frac{c_a + c_b}{2} + \frac{w\,\Delta y}{2\,\epsilon}(c_a - c_y) \qquad (70)$$

in which $(c_y)'$ is the concentration at point y at a time interval Δt later than the concentrations c_a, c_b, and c_y. To use Eq. 70 the values of concentration must be known at $t = 0$ and the increment (Δy) selected; the magnitude of (Δy) to be used will be determined by the accuracy desired. In checking the curves shown in Fig. 7 the writer selected $\Delta\left(\frac{y}{h}\right) = 0.20$ which made $\Delta y = 8.32$ cm, and $\Delta t = 3.94$ sec.

Eq. 70 cannot be used for the points at the top and bottom since for these points the special boundary conditions that apply must be used when substituting into Eq. 17*c*. Thus, for the curves shown in Fig. 7, for the bottom point, $\frac{\partial c}{\partial y} = 0$. The increment equation for the bottom point then becomes:

$$(c_0)' = c_1 \qquad (71)$$

in which, again, $(c_0)'$ is the concentration at the bottom at $(t + \Delta t)$ and c_1 is the concentration at a point (Δy) above the bottom at time t.

For the top point, since $\frac{\Delta c}{\Delta y} = \frac{- w\, c_h}{\epsilon}$, the increment equation is:

$$(c_h)' = c_{h-1} - c_h \left[\frac{w\, \Delta y}{\epsilon} + \frac{w^2\, (\Delta y)^2}{2\, \epsilon^2} \right] \qquad (72)$$

in which c_{h-1} is the concentration at a point (Δy) below the top at time t. Using Eqs. 70, 71, and 72, the values of sediment concentrations were computed for points having relative values of $\frac{y}{h}$ equal to 0, 0.2, 0.4, etc., for fourteen time increments or a total time of $14 \times 3.94 = 55.16$ sec. The values obtained checked the curve given in Fig. 7 for $t = 54$ sec as closely as this plotting could be read. For $t = 24$ sec the check was not quite as good, but adequate for practical purposes.

For the curves shown in Fig. 11, Eqs. 70 and 72 still apply; however, the bottom conditions are different since in this case $\frac{\partial c}{\partial y_0} = - \frac{w\, A}{\epsilon}$, in which A is the concentration at $y = 0$, $t = \infty$. The increment equation to use for obtaining values for the bottom point is then:

$$(c_0)' = c_1 + A \left[\frac{w\, \Delta y}{\epsilon} - \frac{(\Delta y)^2\, w^2}{2\, \epsilon^2} \right] \qquad (73)$$

in which c_1 is again the concentration at a point (Δy) above the bottom at time t. To obtain concentration values for the conditions shown in Fig. 11 the value of Δy is taken as 0.2 h, or 9.04 cm, and since $\Delta t = \frac{(\Delta y)^2}{2\, \epsilon}$ the values of c are obtained for time increments of 9.87 sec. Computations for twelve increments or 118.5 sec checked the values given for $t = 120$ sec in Fig. 11 fairly well. Again, the check is not as good for the first few time increments.

One of the principal advantages in solving a differential equation by numerical computation is that complex boundary conditions can be handled,

and, also, in the present problem, for instance, the value of ϵ could be varied if necessary with y or t without unduly complicating the problem. Numerical methods, although inelegant and sometimes laborious, nevertheless have proved to be tremendously powerful tools in many fields of work and have provided answers to problems that could not have been solved by more formal methods.

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(7) **"Experiments on the Transportation of Sediments in Suspension,"** by V. A. Vanoni, Am. Geophysical Union, 1941. (*a*) p. 613.

(12) **"The Relation of Suspended to Bed Material in Rivers,"** by E. W. Lane and A. A. Kalinske, *Transactions*, Am. Geophysical Union, Section of Hydrology, 1939, p. 637.

(20) **"Measurement of Evaporation from Land and Water Surfaces,"** by C. W. Thornthwaite and Benjamin Holzman, *Technical Bulletin No. 817*, U. S. D. A., 1942.

(21) **"Note on the Variation of Wind with Height in the Layer near the Ground,"** by C. W. Thornthwaite and Maurice Halstead, *Transactions*, Am. Geophysical Union, 1942, pp. 249–255.

(22) **"Wind Gradient Observations,"** by C. W. Thornthwaite and Paul Kaser, Am. Geophysical Union, 1943.

(23) **"Experiments on Eddy Diffusion and Suspended Material Transportation in Open Channels,"** by A. A. Kalinske and C. L. Pien, *Transactions*, Am. Geophysical Union, 1943 (in press).

(24) **"A Study of Transportation of Fine Sediments by Flowing Water,"** by C. H. Hsia, thesis presented to the Department of Mechanics and Hydraulics, Univ. of Iowa, Iowa City, in 1943 in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SLUDGE DRYING DEVELOPMENTS AT CHICAGO, ILL.

Discussion

BY PAUL HANSEN

PAUL HANSEN,[3] M. AM. SOC. C. E.[3a]—To the extent that it details the problems resulting from sludge drying and incineration, this paper is of exceptional value. Such papers as this lead to a better understanding of sludge disposal by burning, and aid greatly in advancing current practice.

An installation whereby sludge, whether raw or digested, from the activated sludge process or from sedimentation only can be handled for the production of salable fertilizer, for steam production, and for the destruction of the sludge at high temperatures represents an almost ideal solution of the sludge problem, provided it can be done reliably and economically.

The Chicago Sanitary District has been a pioneer in this field and has developed the process to a point of practicability and reasonable economy. On the other hand, many minor difficulties are still unsolved as suggested by the author. This is but natural, when the complex character of sewage sludge, and especially raw sewage sludge, is considered. Moreover, sewage sludges vary in different installations. In some cases, as at Chicago and a number of other cities in the Middle West, sewage contains large quantities of packing-house wastes introducing the grease problem. In other places, paper-mill wastes influence the character of the sludge. However, paper-mill wastes retard digestion because of their carbonaceous character; but they facilitate heat disposal. Some plants providing partial treatment by sedimentation and partial digestion, as at Buffalo, N. Y., produce a sludge low in fertilizer and heat value. Thus, the sludge varies greatly in its adaptability to incineration and in its value as a fuel and as a fertilizer.

There is also an interrelation, both economic and functional, between heat destruction of sludge, on the one hand, and prior treatment by storage and digestion, on the other. This relationship has not been exploited completely.

NOTE.—This paper by Lloyd M. Johnson, Esq., was published in September, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by C. W. Gordon.

[3] (Greeley & Hansen), Chicago, Ill.

[3a] Received by the Secretary September 11, 1943.

At Buffalo, a cost analysis of three projects, comprising (1) incineration of raw sludge, (2) incineration of stored sludge (partly digested), and (3) incineration of fully digested sludge, indicated an economic preference for No. (2).

The experiences at Buffalo parallel somewhat the experiences at Chicago. At Buffalo, the cages in the flash driers wore considerably in the beginning. It is clear that the replacement of the bars in the cages must be considered as a regular maintenance problem. The labor and cost involved, however, have been substantially reduced by the utilization of sleeves of especially hard metal that would slip over the bars.

Another difficulty encountered was the wearing of bends in the piping of the ash handling and dry sludge feeding systems. Here again is another element that requires regular maintenance. Difficulties have been overcome by the use of a steam ejector, instead of a vacuum pump, with longer bends and couplings that can be taken apart readily, and with concrete jackets at the bends. Wear in the cyclones was encountered, as at Chicago, and this has been remedied by the use of renewable cement linings.

In a broad sense, the process of sludge handling by incineration, with provision for diverting the material of fertilizing value and with provision for utilizing heat, is in about the same status as the automobile was in the early years of its general use. It works, but it requires considerable tinkering with every hundred miles or so. Little by little the defects and limitations have been revealed and overcome. Today the automobile is reliable for thousands of miles and requires no more attention than systematic oiling and greasing. In the field of sludge incineration, it may now confidently be expected that the minor problems will be overcome and that longer life and more reliable service will result.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

PRIMARY RÔLE OF METEOROLOGY IN FLOOD FLOW ESTIMATING

Discussion

BY MESSRS. C. H. EIFFERT, AND GAIL A. HATHAWAY

C. H. EIFFERT,[10] M. AM. SOC. C. E.[10a]—Much new information has been collected in this field during the past few years, perhaps largely because of the increased interest in aviation. The U. S. Weather Bureau and Mr. Bernard are to be commended for the progress in this direction.

When the Miami Conservancy District investigations were begun about 1913, little information was available outside of the Weather Bureau records of precipitation, temperature, and wind movement. Therefore it is quite gratifying to note that what Mr. Bernard calculates to be the maximum possible storm that could occur, in the area where the 1913 flood actually did occur, agrees quite closely with what was decided on as the maximum possible when the Miami Conservancy District studies were made.

In connection with his calculations Mr. Bernard reaches the following conclusion (see heading, "Procedure in Developing the Maximum Possible Storm for an Individual Basin"): "* * * the March, 1913, storm rainfall could have been 28% greater or 128% of the observed depths."

The following paragraph is quoted from "Storm Rainfall of Eastern United States," Part V, *Technical Reports of the Miami Conservancy District* (34),[10b] under the heading, "Reasons for Choosing as Basis of Design a Flood 40 Per Cent Greater than that of March 1913":

> "The extensive investigation of storms in the eastern United States leads to the belief that the March, 1913, flood was one of the great floods of centuries in the Miami Valley. In the course of three or four hundred years, however, a flood 15 or 20 per cent greater may occur. It is not believed that a flood will ever occur which is more than 20 or 25 per cent in

NOTE.—This paper by Merrill Bernard, M. Am. Soc. C. E., was published in January, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1943, by Edgar Dow Gilman; April, 1943, by Messrs. Clarence S. Jarvis, Ivan E. Houk, W. G. Hoyt, and L. R. Beard; and June, 1943, by Messrs. Robert E. Kennedy, and L. K. Sherman.

[10] Chf. Engr. and Gen. Mgr., The Miami Conservancy Dist., Dayton, Ohio.

[10a] Received by the Secretary June 17, 1943.

[10b] Numerals in parentheses, thus: (34), refer to corresponding items in the Bibliography (see Appendix 1 in the paper), and at the end of discussion in this issue.

excess of that of March 1913. There is a factor of ignorance, however, which should be provided against, and the only way this could be done was arbitrarily to increase the size of maximum flood provided for. If longer records were available a closer estimate could be made, but in planning works on which the protection of the Miami Valley depends, it was necessary to go beyond human judgment. This was done on all the other phases of the design, and it was believed that it would not be good engineering practice to stop at individual judgment on this phase. The planners of the project felt that they must be able to say that the engineering works are safe in every respect. For this reason provision was made for a flood nearly 40 per cent greater than that of March 1913. This is 15 or 20 per cent in excess of what is believed to be the greatest possible flood that will ever occur."

In this discussion the writer cannot refrain from referring again to the possibility of Weather Bureau records being kept in such a way as to be more useful and less difficult to interpret by engineers making hydrological studies. The principal Weather Bureau stations record their precipitation from midnight to midnight; many observers take evening readings; many others make their observations in the morning; and some are recorded from noon to noon. When storm rainfall maps and studies involving such records are prepared, it is often very difficult and sometimes impossible to determine on which day a considerable part of the rainfall actually occurred. This is particularly true in connection with storms of short duration. Why would it not be possible to change this system so that all readings are taken at approximately the same time of day? Most observers could make either morning or evening observations, and those who could not read at the required time could be replaced gradually. Obviously it is not practical for the ordinary observer to make his observations at midnight. However, the midnight stations have recording gages so that their records are adaptable to any time and could be used in connection with either morning or evening records. Originally, the Weather Bureau rainfall observations were not intended for this purpose, being thought of mainly in connection with agriculture. The extensive use of these records in recent years by engineers and hydrologists would seem to be of sufficient importance to justify such a change.

On the first page of his paper, the author refers to time-area-depth values for about 150 storms. Throughout the remainder of the paper these values are referred to as area-duration-depth values. On the Miami Conservancy District work the term time-area-depth was used. Although the word "duration" is slightly more specific in its meaning, either expression is readily understandable. "Time-area-depth" would seem to this writer to be preferable for the sake of its euphony and greater ease of oral expression.

GAIL A. HATHAWAY,[11] M. AM. SOC. C. E.[11a]—Excellent progress is being made in this extremely important phase of flood control as well as in connection with all problems involved in the development and utilization of water resources in the United States. The paper is a comprehensive report of this progress. The investigation of the meteorological and hydrological aspects of major

[11] Hydr. Engr., War Dept., Office, Chf. of Engrs., Washington, D. C.

[11a] Received by the Secretary July 23, 1943.

storms by the Hydrometeorological Section of the U. S. Weather Bureau has furnished the engineer with a much clearer picture of the probable limiting rates of storm rainfall.

In discussing the general subject of storm studies it may be of interest to review briefly some of the events leading to the present work of the Hydrometeorological Section of the Weather Bureau which has proved so valuable in the flood control work of the Corps of Engineers. Shortly after the great storm of May 30–31, 1935, in the headwaters of the Republican River basin, the writer was engaged on a study to determine a spillway design flood for the Kingsley Dam (previously the Keystone Dam) on the North Platte River near North Platte, Nebr. The question immediately arose as to whether it would be reasonable to expect a storm similar to that of May, 1935, to occur over the drainage area of the North Platte River above Kingsley Dam, which is about 150 miles north of the Republican River headwaters. At that time E. J. Minser (chief meteorologist, Transcontinental and Western Air, Inc.) and the writer made a meteorological analysis of the storm to ascertain the feasibility of transposing it to the North Platte River drainage area. After careful study the conclusion was reached that there was no good reason, from the meteorological or physiographical viewpoint, why the storm could not have occurred over the North Platte basin above Kingsley Dam. Similar meteorological studies were made by the writer in 1935 and 1936, while he was employed by the U. S. Engineer Department, for the Sardis Dam on the Little Tallahatchie River in Mississippi, the Bluestone Dam on the New River in West Virginia, and the Possum Kingdom Dam on the Brazos River in Texas.

During the summer of 1936, arrangements were made by the Corps of Engineers for the assignment to the Office, Chief of Engineers, of the late Wilson Reed, Jr., to conduct meteorological studies of selected river basins. However, the demand from the field offices of the Engineer Department for meteorological studies was so great that, later in 1937, a group was set up in the Weather Bureau to make the studies. As stated by the author, this group had been working for, and in cooperation with, the Engineer Department in the review of the meteorological aspects of storms and the determination of maximum storm rates for specific drainage basins.

The first line of attack in any hydrological problem involving runoff is a thorough analysis of rainfall data. The author has reviewed the historical background of storm records and the early investigation and analysis of storm rainfall data. The early studies of storm rainfall are extremely valuable, but the present-day hydrologic technique requires that rainfall intensities be broken down into periods of less than 24 hr. Analysis of a large number of major storms in the United States shows that the maximum rainfall intensities usually occur in periods of 12 hr or less, interspersed with periods of minor rainfall rates. It is considered essential, therefore, that maximum rainfall quantities be determined for periods of 6 hr and in some cases for shorter time intervals. Because of the importance of a thorough analysis of storm rainfall the Engineer Department, in 1937, organized a program for the study of several hundred major storms covering the entire continental area of the United States. The program also includes the analysis of current storms of major proportions.

These recent storm studies are extremely valuable because of the existing network of recording rainfall stations, and the large number of upper-air observations made by means of pilot balloons and "radiosondes."

The study of major storms by the division and district offices of the Engineer Department has been divided into Part I and Part II. Part I of the study consists of the compilation of basic precipitation data, preparation of mass rainfall curves, and isohyetal maps. In addition to published records of the Weather Bureau, Part I includes all available precipitation data and miscellaneous information on storms obtainable from the manuscripts of original records, files of municipal agencies, newspapers, testimony of witnesses, and similar sources. In the event of a major storm, representatives of district offices conduct a field survey in the areas of excessive rainfall to obtain a record of rainfall measurements secured by local residents. The location and method of obtaining these "bucket" or "washtub" records are analyzed carefully on the ground and every effort is made to classify them as to their reliability. Part I of the study, assembled and organized by the various district offices, is reviewed by the Hydrometeorological Section of the Weather Bureau and the mass rainfall curves are correlated with the meteorological analysis of the storm.

Upon completion of the review by the Hydrometeorological Section, Part I is returned to the originating district office for completion of Part II of the study. The preparation of Part II involves the following steps:

(*a*) Preparation of the final total-storm isohyetal map based on the selected storm period, or delineation of zones on the isohyetal map and grouping of rainfall stations by zones;

(*b*) Tabulation of contemporaneous rainfall quantities as scaled from mass rainfall curves, for periods usually increasing by 6-hr increments;

(*c*) Tabulation of absolute maxima rainfall quantities for durations of 6, 12, 18, and 24 hr for stations within the zones of excessive rainfall intensity;

(*d*) The computation of mass rainfall curves representing the average depth of rainfall over selected areas of the storm; and

(*e*) The computation of maxima duration-depth-area data for various combinations of zones.

The final isohyetal map showing the zone subdivisions and the duration-depth-area curves for a typical major storm are shown in Figs. 24 and 25. In this case, mass rainfall curves were prepared for stations underscored with a single line; and recording-gage records were available for stations underscored with a double line. In most cases, rainfall values of less than 3 in. were interpolated from published records without constructing mass curves. Hence they are approximate. For the purpose of clarity, the odd numbered isohyets above 4 in. have been deleted from the isohyetal map in Fig. 24. All of the isohyets (that is, 1 to 21 in.) were used in computation of the depth-area data in Fig. 25. Pertinent data sheets similar to that shown in Fig. 26 are prepared for each storm study for use of the various district offices in determining whether the completed storm studies have a direct bearing on problems in their

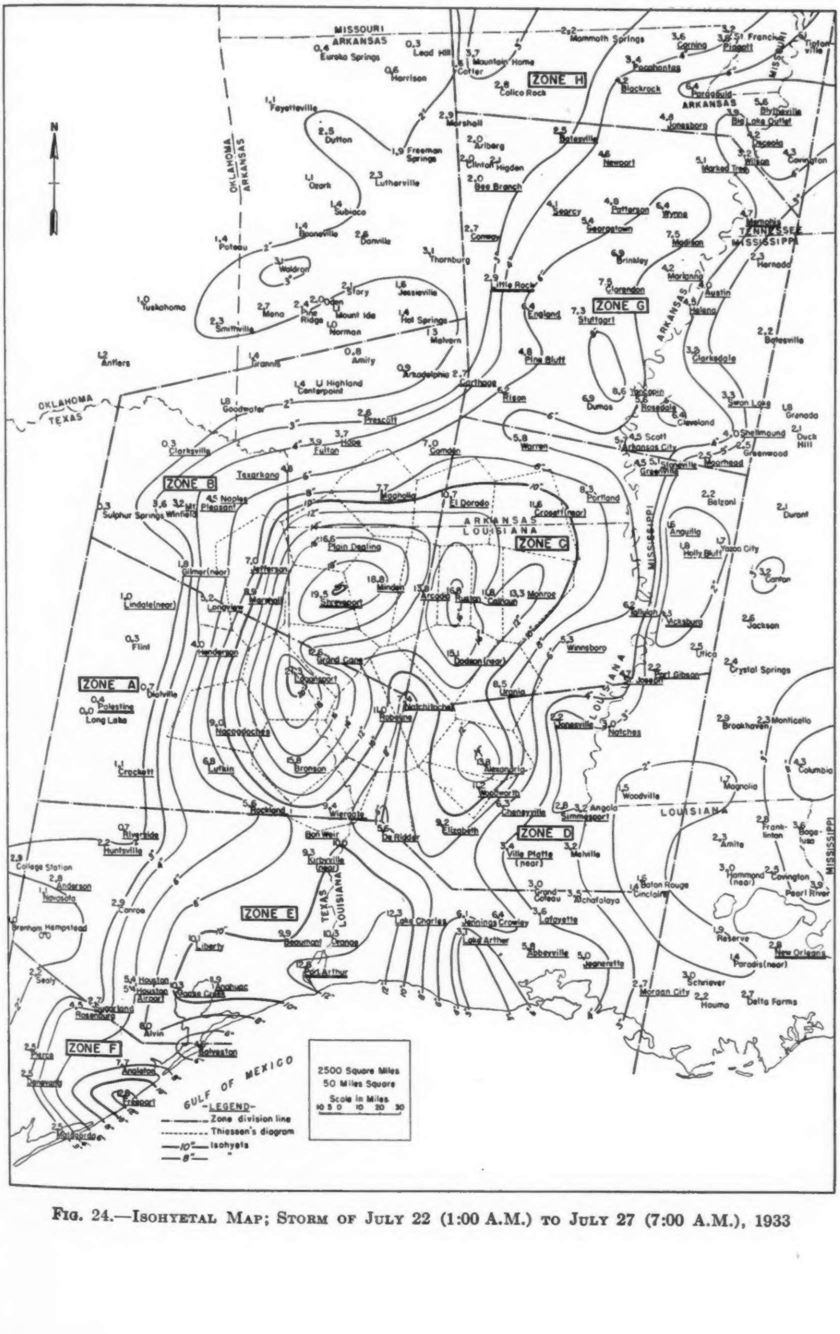
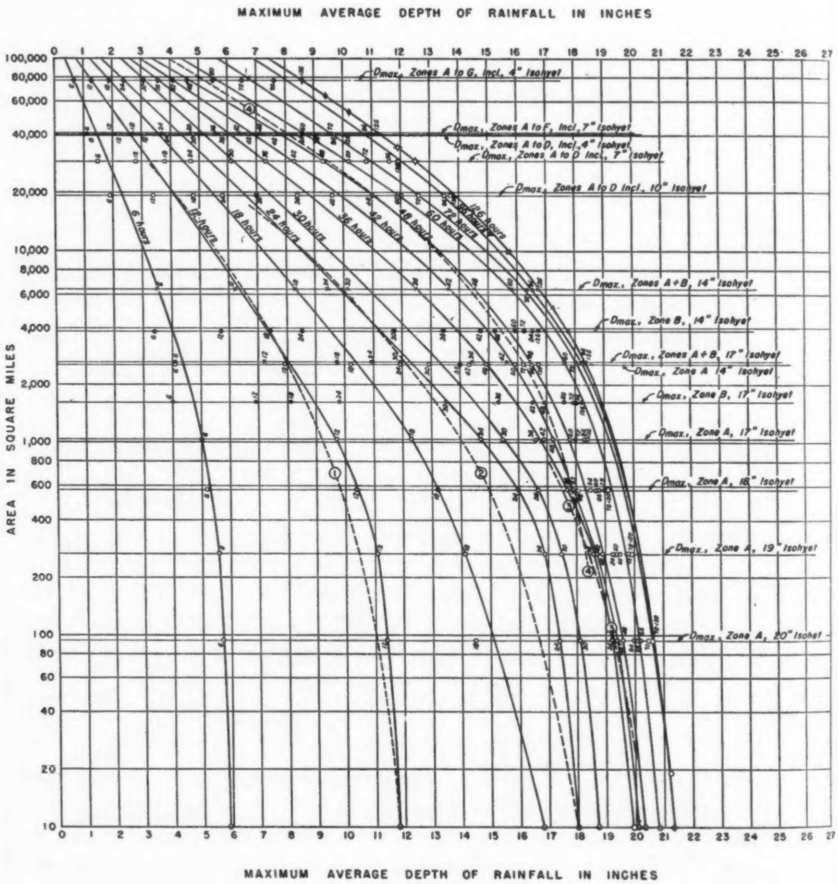


FIG. 24.—ISOHYETAL MAP; STORM OF JULY 22 (1:00 A.M.) TO JULY 27 (7:00 A.M.), 1933

respective offices. The sheets also serve as an index to the data assembled in each study.

At the beginning of the storm study program, 1,350 storms were assigned to district offices of the Engineer Department. However, it is not expected that all of the storms under study will be completed to the point of computing 6-hr rainfall quantities and preparing duration-depth-area data for short-time



EXPLANATION

"D_{max}, Zones A + B, 14″ isohyet" refers to maximum depth-duration values computed for the area within zones A + B and the 14 in. isohyet.

Curves shown by solid lines represent final maximum depth-area curves.

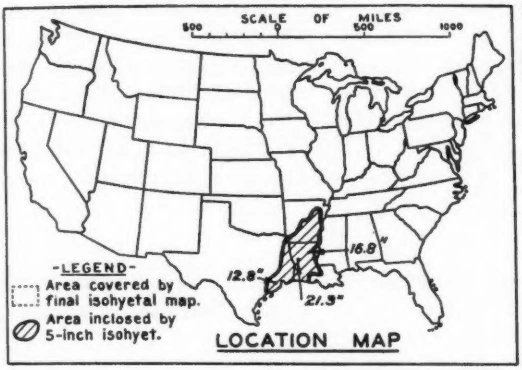
Depth-area curves shown in broken line (– – –) were computed (for comparison only) by planimetering isohyetal maps for following periods. (1) 12 hr ending 1 A 24, (2) 24 hr ending 7 A 24, (3) 48 hr ending 7 P 24, (4) 48 hr ending 1 P 25.

Plotted points represent maximum average depth of rainfall, over the area designated, within the number of hours indicated by figures beside the points.

Identical legends were used for all depth-duration curves within the same zone or combination of zones.

FIG. 25.—MAXIMUM DEPTH-AREA CURVES; STORM OF JULY 22 TO 27, 1933

STORM STUDIES – PERTINENT DATA SHEET



Storm of July 22-27 1933
Assignment L.M.V. 2-26
Location Ark., La. & Tex.
Study Prepared by: Office, Chief of Engineers

Part I Reviewed by H. M. Sec. of Weather Bureau, - -
Part II Approved by Office, Chief of Engineers for Distribution of Factual Data, 5-30-40
Remarks: Part II used as Supplement C, Engineer Bulletin R. & H. No. 10, 1938, E. D. 7390.

DATA AND COMPUTATIONS COMPILED

PART I

Preliminary isohyetal map, in 1 sheet, scale 1: 2,500,000

Precipitation data and mass curves: (Number of Sheets)

Form 5001-C (Hourly precip. data)	3
Form 5001-B (24-hour " ")	-
Form 5001-D (" " " ")	-
Miscl. precip. records, meteorological data, etc. (5001 WB)	322
Form 5002 (Mass rainfall curves)	37

PART II

Final isohyetal maps, in 1 sheet, scale 1: 1,000,000

Data and computation sheets:

Form S-10 (Data from mass rainfall curves)	5
Form S-11 (Depth-area data from isohyetal map)	2
Form S-12 (Maximum depth-duration data)	8
Maximum duration-depth-area curves	1
Data relating to periods of maximum rainfall	2

MAXIMUM AVERAGE DEPTH OF RAINFALL IN INCHES

Area in Sq. Mi.	Duration of Rainfall in Hours										
	6	12	18	24	30	36	48	60	72	96	126
10	5.9	11.8	16.8	18.0	18.7	19.9	20.3	20.8	21.3	21.3	21.3
100	5.7	11.4	15.1	17.4	18.1	19.2	19.6	20.2	20.6	20.7	20.7
200	5.6	11.2	14.4	17.1	17.7	18.8	19.2	19.8	20.2	20.3	20.3
500	5.4	10.6	13.4	16.2	16.9	17.9	18.4	19.4	19.7	19.9	19.9
1,000	5.0	9.7	12.4	14.8	15.6	16.8	17.8	18.8	19.2	19.3	19.4
2,000	4.6	8.5	11.0	13.0	14.0	15.5	16.9	18.0	18.4	18.6	18.7
5,000	3.8	6.8	8.9	10.5	11.5	13.2	15.1	16.4	16.8	17.1	17.3
10,000	3.1	5.5	7.2	8.5	9.5	11.1	13.3	14.3	14.9	15.4	15.7
20,000	2.3	4.1	5.4	6.5	7.4	8.6	10.9	11.8	12.4	13.3	13.7
50,000	1.2	2.2	3.0	3.7	4.5	5.3	6.8	7.7	8.7	9.9	10.4
100,000	0.4	0.8	1.2	1.6	2.3	2.6	3.4	4.4	5.6	6.9	7.4

FIG. 26.—PERTINENT DATA SHEET OF STORM STUDIES

intervals. Only the major storms in each section of the United States with critical rates for different intervals of time will be analyzed completely.

In connection with the presentation of duration-depth-area data by the author in Table 1 it appears desirable to point out that the depth-area relationships for the August 6–7, 1935, storm in Ohio, and the July 4–5, 1939, storm in Kentucky were derived largely from unofficial or so-called "bucket" observations; but no footnote has been added to indicate that the results were obtained from other than official observations. A brief footnote in such instances, concerning the source and relative accuracy of the data, would seem appropriate.

Bibliography.—

(34) **"Storm Rainfall of Eastern United States,"** *Technical Reports of the Miami Conservancy District*, Pt. V., Revised Ed., p. 335 (1917 Ed., p. 273).

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

FLOW AROUND BENDS IN STABLE CHANNELS

Discussion

BY I. NELIDOV

I. NELIDOV,[30] M. AM. SOC. C. E.[30a]—So far as it is known to the writer, there has been no clear exposition of the mechanics of flow in curved channels. Professor Mockmore's accomplishment therefore is that he produced an analysis

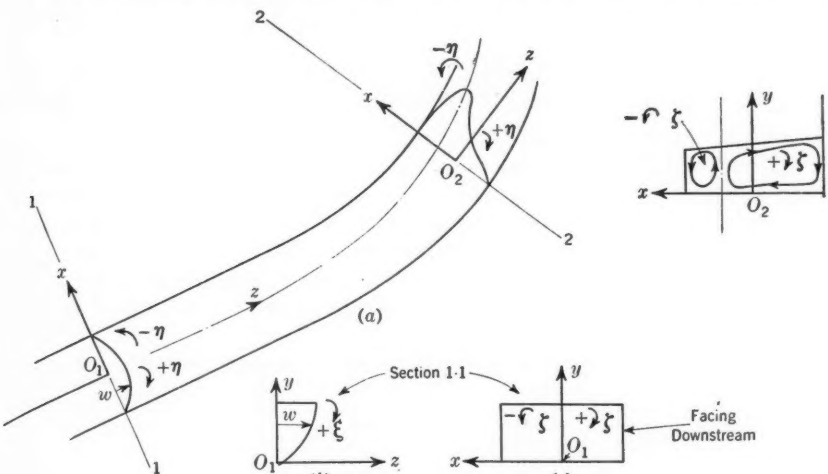

FIG. 20

of the components of the velocities on three orthogonal planes within the body of the current.

The writer wishes to illustrate this problem from the standpoint of mechanical analogy to gyroscopic motion. From the theory of gyroscopic[31] motion,

NOTE.—This paper by C. A. Mockmore, M. Am. Soc. C. E., was published in March, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Messrs. J. M. Robertson, and Joe W. Johnson and M. A. Selim.

[30] Senior Engr., U. S. Engr. Dept., Sacramento, Calif.

[30a] Received by the Secretary August 9, 1943.

[31] "Engineering Mechanics," by S. Timoshenko and D. H. Young, 1937 Ed., Pt. II, Dynamics, p. 255.

if a body is acted on by two orthogonal couples, there will exist, as a result of this action, a third couple orthogonal to the first two couples. Consider a straight part of a channel as shown in Fig. 20(*a*), Section 1–1. The approximate velocity distribution in plan and vertical cross section is known and shown in Figs. 20(*b*) and 20(*c*). This distribution of velocities results in the couples η and ζ. Since these couples have a negative and a positive sign, the water will flow so that, in a straight reach of channel, there will be two spirals, one clockwise and the other counterclockwise.

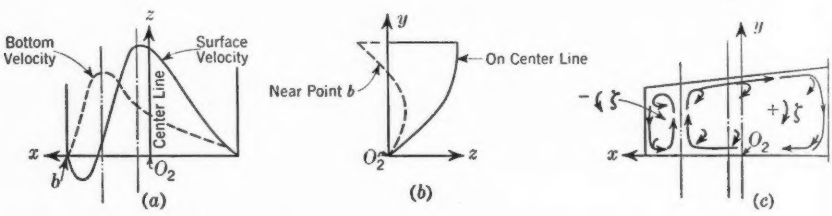

Fig. 21

It may be noted here that M. Möller[32] quotes experiments in the Ruhr River which indicated the reverse direction of spirals from the one in Fig. 20(*c*). However, he maintains that two spirals will exist in a straight reach of channel.

In the curved reach of channel, the relations will be as indicated in Fig. 20(*d*), which is in general agreement with Fig. 5(*b*). If there are reversals in distribution of the velocity w either in plan or in a vertical section, there will be, as a result of this condition, some additional couples which, if of opposite sign, may create "dead" water. Therefore, before the direction of spiral rotation is established, a rather complete picture of the distribution of velocity w should be obtained. Considering a reversal of velocities w near the point b in the flow indicated in Section 2–2 in Fig. 20 near the surface (as shown in Fig. 5(*c*)) and no reversal at the bottom, the rotations as viewed facing downstream will be as shown in Figs. 21(*a*) and 21(*b*).

Professor Mockmore has presented a very clear picture of mechanics of spiral flow.

[32] "Hydraulic Laboratory Practice," by J. R. Freeman, article by M. Möller, 1929 Ed., p. 70.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

PHYSICAL PROPERTIES THAT AFFECT THE BEHAVIOR OF STRUCTURAL MEMBERS

Discussion

BY A. A. EREMIN

A. A. EREMIN,[31] Assoc. M. Am. Soc. C. E.[31a]—The physical properties of steels affecting the design of structures may be determined from diagrams expressing stress-strain relations for tests on steel. Sufficient data are not yet available to draw the diagrams for various types and shapes. However, considering the similarity of some characteristic features of steels, the diagrams available at present are sufficient to determine the approximate influence of the

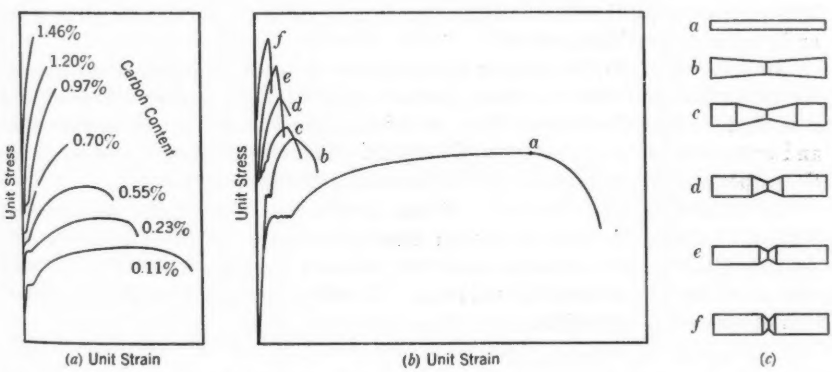

FIG. 2.—RELATIVE STRESS-STRAIN DIAGRAMS FOR STEEL

physical properties of steels on the behavior of structures. Interesting diagrams of this stress-strain relation have been presented by S. Timoshenko.[32]

NOTE.—This paper by Wilbur M. Wilson, M. Am. Soc. C. E., was published in December, 1942, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1943, by Jonathan Jones; May, 1943, by A. B. Kinzel; June, 1943, by Messrs. Almon H. Fuller; and September, 1943, by Messrs. R. L. Moore, Leon S. Moisseiff, and Fred L. Plummer.

[31] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[31b] Received by the Secretary August 13, 1943.

[32] "Strength of Materials," by S. Timoshenko, Pt. II, 2d Ed., Van Nostrand, p. 553.

In Fig. 2(*a*) stress-strain diagrams for steels with various carbon content are shown. The ductility is decreased with an increase of the carbon content. The alloy steels are characterized by a carbon content varying from 0.25% to 0.50%. Therefore, Fig. 2(*a*) makes it possible to determine an approximate relation between the plastic flow of alloy steels and A7 steel. These curves

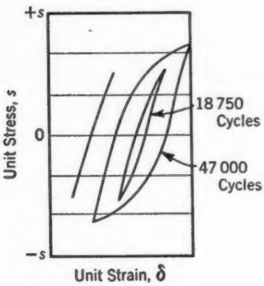

FIG. 3

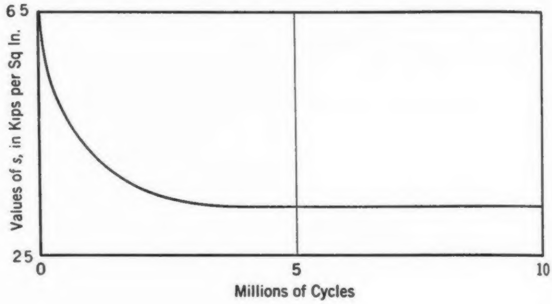

FIG. 4

prove the author's statement that, where plastic flow is desirable, the low-alloy steels or A7 steel should be used. Fig. 2(*b*)[33] shows relative stress-strain diagrams for steel members with reduced mid-sections such as those in Fig. 2(*c*). Members in which notches are cut, or members with variable cross section, lose their ductility although their ultimate strengths are increased.

The capability of steel to harden under repeated loading is shown in Fig. 3. It is obvious that the alternate loading in a member may raise its yield point and produce a stiffening effect.

In conclusion (1), the author states that structural members should be designed for known forces and then checked against fatigue failure. From Fig. 4 it is obvious that the fatigue effect on stress in the members may be estimated and considered as a special type of loading. Therefore, it is possible to design the structure for a combined loading including the fatigue effect.

In conclusion (2), Professor Wilson states that the plastic deformation permits the redistribution of uneven localized stresses. From Fig. 2 it is evident that the plastic deformation at the reduced sections, where the localized stresses occur, is considerably reduced. Therefore, its effect on the redistribution of stresses is negligible.

[33] *Der Stahlbau*, June 19, 1936, p. 109.

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DISCUSSIONS

RÔLE OF THE LAND DURING FLOOD PERIODS

Discussion

By CHARLIE M. MOORE

CHARLIE M. MOORE,[24] JUN. AM. SOC. C. E.[24a]—The actual application of a method for determining the measured effect of land, soil moisture, and vegetal cover on runoff and infiltration from natural rainfall is presented in this enlightening and interesting paper.

In discussing Step 1 (heading, "Methodology") the author has applied rainfall data collected by the writer at Lindale and Mount Pleasant, Tex., during the 5-yr period from 1935 to 1940. At each of these locations the federal government installed three Ferguson recording gages, and five standard gages within demonstration project areas of some 23,000 acres. The recording gages were supervised by junior engineers, whereas it was necessary to rely upon co-operating farmers to measure and record the precipitation collected by the standard gages. A diligent effort was made to keep the recording gages in good operating condition and to obtain records as accurate as possible for all gages. Cooperative observers were selected carefully in almost every instance before installing the gages. However, some of the original observers moved or became disinterested after a period of 2 yr or 3 yr. Where they were interested it was not difficult to obtain a satisfactory record that could be depended upon In the monthly collection of data from cooperative observers, it was often necessary to redate the reports and to compare the recorded amounts of the rains with the nearest recording gages. Quite often several hours would elapse before the observer would remember to measure the rain. At times, after small rains, the precipitation would be evaporated before measurements were made. This fact, no doubt, made it difficult to analyze the data at a later date.

As illustrated by the author, the application of the method to drainage basins as large as 831 sq miles gives some valuable information to engineers interested in the determination of maximum runoff to be expected from watersheds, and the reduction that can be expected from the application of the De-

NOTE.—This paper by W. W. Horner, M. Am. Soc. C. E., was published in May, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1943, by L. K. Sherman; and September, 1943, by Messrs. Raphael G. Kazmann, and W. G. Hoyt and W. B. Langbein.

[24] District Engr., SCS, Crockett, Tex.

[24a] Received by the Secretary September 7, 1943.

partment of Agriculture program, as related to erosion control, land use, and management.

The engineers of the Soil Conservation Service, in Region Four, of necessity use the rational method in determining the runoff to be expected for a given frequency of rainfall. This method, which is based on the formula—

$$Q = C\,I\,A \qquad (6)$$

—is used for comparatively small agricultural areas and is limited to a maximum of approximately 1,000 acres. It is realized by engineers that some such method as the author has given is needed to evaluate properly the rôle of the land and the part that it plays in reducing the runoff where proper land use and management are employed. Also, a method similar to that of the paper, which can be applied to larger watersheds, is needed.

In the application of Eq. 6, a constant runoff factor is generally chosen and assigned to certain land-use groups within a soil group. However, it can be seen that such a procedure does not take into account the effect of previous rainfall, type of cover, condition of cover, degree of erosion, or variations in soils within a soil group. The method presented by the author will evaluate most of these factors and their effect upon the runoff.

It is noted in Step 3 that the problem of deriving infiltration-capacity curves for the Blackland soil and various classes of vegetal cover was simplified in that, to all intents and purposes, only one soil type occurred in the basin, and in that the variations in infiltration capacity were primarily those growing out of the difference in vegetal cover and of antecedent conditions. This problem would be very acute in the sandy lands of eastern Texas, and other areas, where some sixty to seventy soil types occur, ranging from deep sands to very heavy, almost impervious, clay soils. Many of these soils occur within very small drainage basins in the eastern Texas area.

The writer agrees with the statement that a system of bookkeeping is needed that will reveal the values of infiltration capacity and of utilizable soil-storage capacity in their occurrence pattern over a considerable period of years. This will help to form a sound hydrologic basis for the evaluation of storm runoff.

In considering the rôle of the land and its effect upon runoff, the writer has found that, by treating drainage basins properly with sound conservation practices, such as terracing, contour cultivation, contour ridges, retiring steep slopes to grass and woodland, controlling gullies with dams, and sloping and sodding gullies, considerable expense can be saved in the capacity of bridges, culverts, spillways, etc., in the lower reaches of the watersheds. These practices help to retain the water on the land for a period sufficient to cause most of it to be absorbed. By proper land use and management in a drainage basin, the reduction of runoff, as illustrated by the author, can well be expected. The mere changing of a system of cultivated rows by a farmer can cause almost total runoff as compared to a very small amount of runoff if the rows are laid off on the contour. The writer has observed that flood flows on drainage basins ranging in size from 100 to 1,000 acres or more—on which the Department of Agriculture program had been applied almost 100%—were reduced consider-

ably. On one of these basins it had been impossible for the county government to construct semipermanent bridges across the main channel without having them overflowed or washed out. County officials have been troubled very little with this problem since the watershed has been treated.

The writer is anticipating the time when sufficient information is compiled and made available to engineers for use in the application of this method to various sizes of watersheds or drainage basins. The author has advanced this method appreciably by the publication of this paper.

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DISCUSSIONS

SEDIMENTATION IN RESERVOIRS

Discussion

BY MESSRS. JOHN W. STANLEY, STAFFORD C. HAPP, AND THOMAS H. MEANS

JOHN W. STANLEY,[5] ESQ.[5a]—Studying the subject of reservoir silting and compiling data required much labor and time. Mr. Witzig is to be commended for his work in attempting to develop a method for predicting the sedimentation rate. It seems quite certain that any energy, time, and money spent in further study of the problem would be many times repaid by more efficient operation and longer life of reservoirs.

Engineers and geologists have given much thought to the relationships among the many factors affecting the transportation and deposition of sediment, with the result that considerable progress has been made. Some facts have been established fairly well, and some promising theories have been advanced. In the process, however, a few conclusions, ideas, and suggestions have been submitted which later were found to be undesirable. Any investigator, in attempting to cover the subject by a perusal of pertinent literature, indeed would accomplish the impossible if he were able to weed out all the unacceptable statements. It is with this thought in mind, and with no intention of detracting from the value of Mr. Witzig's paper, that the writer wishes to call attention to the second paragraph under "Origin and Nature of Silt: Definition of Silt."

The statement in this paragraph that "When the vertical components of turbulence are sufficiently strong, no settling takes place" is somewhat misleading. The author undoubtedly intended the statement to mean that no sediment accumulates on the bottom. The present general belief, as pointed out by William E. Dobbins, Jun. Am. Soc. C. E. (42),[5b] and emphasized by Thomas R. Camp, M. Am. Soc. C. E. (42*a*), is that, except for possibly the very finest particles, there is a constant interchange between particles in suspension

NOTE.—This paper by Berard J. Witzig, Jun. Am. Soc. C. E., was published in June, 1943, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1943, by Joe W. Johnson.

[5] Asst. Engr., Bureau of Reclamation, Denver, Colo.

[5a] Received by the Secretary August 26, 1943.

[5b] Numerals in parentheses, thus: (42), refer to corresponding items in the Bibliography (see Appendix II in the paper), and at the end of discussion in this issue.

and those on the stream bed and that this will be true as long as there is any turbulence. It does not seem proper, therefore, to assume that no actual settlement occurs even when the vertical components of turbulence are very strong, although it is true that there might be no accumulation.

In the next sentence of the same paragraph Mr. Witzig refers to Mr. Einstein and to Alvin G. Anderson and Joe W. Johnson, Assoc. Members, Am. Soc. C. E., for this definition of bed load (6*a*): "* * * that part of the total sediment-load composed of all particles greater than a limiting size * * * whether moving on the bed or in suspension, and includes all bed-material in movement." Later in his paper, however, the author refers to bed load as the material moved by rolling along the bed, as distinguished from the material moving in suspension. The meaning actually used by the author is preferable. Bed load has been defined by a special committee of the Society (43) as "The silt, sand, gravel, or other detritus rolled along the bed of a stream," and long usage rather has fixed this meaning for the term. Although it is true that only material coarser than the "dividing grain size" is moved at a rate that is a function of the discharge, slope, etc., it is not necessarily feasible to ignore the fact that part of this material actually moves as suspended load and part as bed load. As someone has suggested, the natural term to use for material coarser than the "dividing grain size" would be bed-material load. By adopting this term and retaining the term bed load with its old meaning, much confusion may be avoided.

In the last sentence of the same paragraph Mr. Witzig refers to Table 2 for "the rate of settlement (7) of a particle in still water at 50° F." The rates in Table 2 are questionable because those given for all particles except the 1-mm and 0.20-mm sizes are greater than the rates generally found for spheres with a specific gravity of 2.65. The rates are essentially the same as those given in a more extensive list by the late Allen Hazen, M. Am. Soc. C. E. (44). Mr. Hazen gave a table of "hydraulic values" for irregular grains (specific gravity approximately 2.65) of which "the diameters are taken as the diameters of spheres of equal volume." He determined these rates by actual experiment for particles from 1.0-mm to 0.10-mm diameter, by the Wiley formula from 0.02 mm to 0.0001 mm, and by interpolation from a connecting curve for particles between these two grades. His rates computed from the Wiley formula are approximately 200% of the rates given by the Stokes formula for spheres; those for particles 0.02 mm to 0.25 mm are greater, by varying amounts, than the rates for spheres (45); and those for 0.30 mm to 1.0 mm are lower than for spheres. Actually, any irregular particle should settle more slowly than a sphere of equal volume and specific gravity, because the irregular particle presents a greater surface area and hence has a greater resistance to motion than the sphere. In fact, if settling rates are to be used for denoting any apparent dimensions of irregular particles, extreme care must be exercised in selecting the rates for a particular type of material. A few workers besides Mr. Hazen have attempted to determine settling rates for irregular particles, and the results have varied widely.

Arthur Holmes (46) performed some experiments, in which he separated quartz grains by means of rising currents of water. The rates he determined

are compared with the corresponding rates for quartz spheres in Table 6. A. Atterberg (47) determined rates for "fine sands" (Si O_2). He gave rates as the time required for particles to settle 10 cm in water and indicated that decantation or another elutriation process was used in making the determinations. He did not state the water temperature, but experiments on capillarity described in the same paper were performed at an approximate temperature of 17° C. The rates as indicated by his data, with corresponding rates for spheres, are shown in Table 6.

TABLE 6.—COMPARISON OF COMPUTED AND OBSERVED SETTLING RATES
(Units Are Millimeters per Second)

Diameter (mm)	Holmes [a]	Atterberg [a]	Quartz spheres
0.40	47	....	55[b]
0.30	32	....	38[b]
0.25	25	....	30[b]
0.20	20	....	22[b]
0.10	6.7	....	7.1[b]
0.06	....	1.8	2.8[b]
0.05	1.8	....	2.0[b]
0.02	....	0.22	0.32[b]
0.01	0.12	....	0.078[c]
0.006	....	0.028	0.03[c]
0.002	....	0.0035	0.0033[c]

[a] Holmes rates determined at 15° C and Atterberg rates at approximately 17° C. [b] See reference (45) in Bibliography. [c] Stokes' formula.

E. C. J. Mohr (48) gives results of his own determinations and those of four other investigators, including Mr. Atterberg, for particles 0.06-mm diameter and smaller. All values are widely different except those of Messrs. Mohr and Atterberg, whose rates for particles 0.02-mm diameter and smaller are very close. No temperatures are given, however, and the apparent agreement therefore might be better or worse if temperatures were known.

C. K. Wentworth (49) desired to extend his sieve analysis curves into the sedimentation range and for this purpose chose certain "hydraulic values." He commented that his scale "has no precise theoretical justification," but that it approximates "the mean of several published tables by hydraulic values." Since he did not indicate at what temperature the values were to apply, it is impossible to compare them with other data. W. C. Krumbein (50) gives a brief résumé of the work of H. Wadell on settling velocities of nonspherical particles.

The writer has done some investigating on the subject and has found values for particles from about 0.03-mm diameter to 0.12-mm diameter. His values are somewhat lower than those of Messrs. Holmes and Atterberg. In the process of investigating the subject, a method was found which apparently permits reliable determination of values for any given material.

The chart, equation, and table developed by Mr. Witzig for use in estimating the rate of reservoir sedimentation are interesting and should be useful, especially if further refinements can be made as more data become available. Perhaps it would be desirable to change the method of designating the "en-

velopes." For example, it seems probable that, by classifying drainage areas according to average annual rainfall and average susceptibility to erosion, the envelopes might be narrowed and the "regional indexes" possibly changed to indexes according to rainfall and erosion characteristics.

That phase of engineering termed "upstream engineering" by Mr. Witzig is one that has been neglected much too often in the past. The engineer should not be relieved of the guilt of failing to take steps, where possible, to prolong the life of a reservoir by means of erosion control, even though he may be able to prove that the cost of the reservoir will have been repaid by the time its useful life is ended by sediment accumulation. It is to be hoped that in the future the problem of erosion control, as related to reservoir life, will be given more nearly the amount of attention merited by its importance.

STAFFORD C. HAPP,[6] ESQ.[6a]—Sedimentation in reservoirs is a complex problem, and, in his attempt to cover the subject in a single brief paper, Mr. Witzig appears to have given some citations which require further clarification. One such reference concerns the findings of the Soil Conservation Service (SCS) reported (3) in May, 1940.

Mr. Witzig (see heading, "Origin and Nature of Silt: Origin of Sediment") appears to contrast the estimate by Mr. Horton that "of the total erosion in the evolution of a humid drainage basin, probably 1% was initially bank erosion, and 99%, sheet erosion (2)" with the SCS studies which Mr. Witzig states "suggest that existing deposits within the flood plain of a stream are now often more important as immediate sources of sediment than concurrent sheet erosion, * * *." These two estimates apply to different conditions, but the second citation appears to be based on a misunderstanding of the actual findings.

The findings of the SCS originally were based on studies in northern Mississippi, but they have since been tested and confirmed in many places from the southeastern Piedmont to the Great Plains, and in the Ohio and upper Mississippi basins. In all these areas, as a general rule, concurrent sheet erosion is the principal source of reservoir sediment, and stream-bank erosion is of minor importance. As an average for the Central States and Southern States, where present information is most complete, the studies of the SCS would not support an estimate of more than 10% for the proportion of reservoir sediment derived from stream-bank erosion.

The relative importance of bank erosion may be greater in forested or mountainous areas, which have not been studied extensively by the SCS, but only in the Southwestern States (where enlargement of arroyos or ephemeral channels is very active) has bank erosion been found to be a principal source of reservoir sedimentation. This latter region is in arid or semi-arid country and on that account cannot be compared directly with Mr. Horton's estimates.

The SCS research did indicate that stream-bank erosion and gullying were major sources of the relatively coarse alluvial deposits which often cause damage to farm lands on the flood plain, but these sands and gravels seldom form

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[6] U. S. Engr. Office, Ocala, Fla. (formerly Geologist, SCS, Washington, D. C.).

[6a] Received by the Secretary September 17, 1943.

any large part of reservoir deposits in the humid region because there are much greater quantities of silt and clay contributed to the reservoirs from sheet erosion on agricultural lands.

THOMAS H. MEANS,[7] M. AM. SOC. C. E.[7a]—The results of much recent research on sedimentation are brought together in concise form by the author. All the references in Appendix II, except (7) (9) (17) and (19), were published in 1933 or thereafter—the period of greatest reservoir building in the United States. It is no wonder the two activities have coincided because many problems of sedimentation are created by reservoirs, and it will be many years before all of them are solved.

One lack in recent studies of sedimentation rates, as illustrated in Fig. 3 and Table 5, is the scarcity of information as to the quantity of sediment deposited above the reservoir level. Deltas of noticeable size have been built upstream from some reservoirs, and certainly the change in stream grade which is created by the reservoir will affect other streams. Bridges and towns and occupied valleys along channels upstream from reservoirs will be affected, and it is to be hoped records will be kept so that the extent of changes can be determined. The writer has examined a number of reservoirs constructed recently where bridges will be menaced in a few years if the to-be-expected sequence of events follows. Owners of these bridges may be able to establish the cause of the increased menace if records are collected.

There seems to be a growing tendency to summarize all investigations in one formula. This may be helpful in understanding general relations but too often, when such a formula is used, the underlying data are not considered. In the present state of knowledge several of the formulas presented by Mr. Witzig should be used with extreme caution or not at all. For example, Eqs. 2 and 3 are suggested as a short cut to sampling to determine the volume of sediment carried by a stream. A few samples run through this mathematical "mill" might produce results but not such as to inspire much confidence in an engineer who has observed silt-bearing streams in action.

Another example is the "silt-rating curve," also designed as a short cut to long sampling periods. The original paper in which this method was described (17) gives the following values of G_s for two periods on the Red River near Gainesville and the Washita River near Durwood:

Year	Red River near Gainesville	Washita River near Durwood
1936–1937	$1.04 \times 10^{-8} Q^2$	$9.2 \times 10^{-8} Q^2$
1938–1939	$3.85 \times 10^{-8} Q^2$	$2.7 \times 10^{-7} Q^2$

The differences between these two years (about 1 to 3) are too large to give one much confidence in this method of determining the flow of silt. One of the first things learned by an engineer, studying silt, is that the silt percentage does not always change as the flow changes.

[7] Cons. Engr., San Francisco, Calif.

[7a] Received by the Secretary September 24, 1943.

The distinction between suspended silt and bed load still seems to be a matter of controversy. No phase of sedimentation has received more study, and today no phase seems to be in a greater state of uncertainty. There are many bed-load formulas but none seems to be valuable for all conditions. The writer was present many times when G. K. Gilbert was conducting his classic experiments at the University of California at Berkeley, and ever since has lived in hopes that someone would produce a usable method of determining the movement of sediment along the stream or canal bed. So far no such method has come to his attention. Bed load becomes very important in any study of sedimentation in reservoirs. For example, the Arkansas River is a great carrier of sand, as the manager of any canal diverting from it knows. What will be the result of Caddoa Reservoir on the movement of this sand and how will the bridges across the stream and the town of Lamar, Colo., be affected? There is an opportunity to collect much usable data from these new reservoirs, and it is to be hoped the Army Engineers will collect information systematically on changes in the bed upstream from reservoirs and of the delta formations that will build up at the point where the flow is checked.

The stream downstream from reservoirs, cleared of sediment, also will create new conditions. The Bureau of Reclamation is making very careful surveys of the Colorado River below Boulder Dam, as revealed by its Eighth Annual Report.

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(3) **"Some Principles of Accelerated Stream and Valley Sedimentation,"** by Stafford C. Happ, Gordon Rittenhouse, and G. C. Dobson, *Technical Bulletin No. 695*, U.S.D.A., May, 1940.

(6) **"A Distinction Between Bed-Load and Suspended Load in Natural Streams,"** by H. A. Einstein, Alvin G. Anderson, and Joe W. Johnson, *Transactions*, Am. Geophysical Union, 1940, pp. 628–633. (*a*) p. 632.

(7) **American Civil Engineers' Handbook,** by Mansfield Merriman, Section 14, by Allen Hazen, John Wiley & Sons, Inc., 5th Ed. 1930, p. 1474.

(17) *Annual Report,* Chf. of Engrs., U. S. Army, Pt. 4, Vol. 8, 1904, pp. 3803–3818.

(42) **"Effect of Turbulence on Sedimentation,"** by William E. Dobbins, *Proceedings*, Am. Soc. C. E., Vol. 69, February, 1943, p. 238. (*a*) Discussion by Thomas R. Camp, June, 1943, p. 978.

(43) **"Standard Symbols and Glossary of Terms Used in Hydraulics as Applied to Irrigation,"** a report of the Society's Special Committee on Irrigation Hydraulics, 1933.

(44) **"On Sedimentation,"** by Allen Hazen, *Transactions*, Am. Soc. C. E., Vol. LIII (1904), p. 63.

(45) **"Terminal Velocity of Fall—Quartz Spheres in Air and Water,"** a chart by U.S.D.A., Cooperative Laboratory, SCS, California Inst. of Tech., Pasadena.

(46) **"Petrographic Methods and Calculations,"** by Arthur Holmes, Thomas Murby & Co., London, England, May, 1921, p. 207.
(47) **"Die rationell Klassifikation der Sande und Kiese,"** by A. Atterberg, Chemiker Zeitung, Johrgang 29, 1905, p. 198 (translated by I. B. Hosig).
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(49) **"Methods of Mechanical Analysis of Sediments,"** by C. K. Wentworth, Univ. of Iowa (Iowa City) Studies in Natural History, Vol. 11, 1925–1927, p. 41.
(50) **"Manual of Sedimentary Petrography,"** by W. C. Krumbein and F. J. Pettijohn, D. Appleton-Century Co., Inc., 1938, pp. 104–107.